Iowa State University
Digital Repository

1988

The selection of technological forecasting models in life analysis

Hyun-Seung Oh
Iowa State University

Follow this and additional works at: https://lib.dr.iastate.edu/rtd

Part of the Industrial Engineering Commons

www.manaraa.com

Order Number 8825430

The selection of technological forecasting models in life analysis

Oh, Hyun-Seung, Ph.D.

Iowa State University, 1988

U·M·I

The selection of

technological forecasting models

in life analysis

by

Hyun-Seung Oh

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Department: Industrial Engineering
Major: Engineering Valuation

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1988

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I. INTRODUCTION

As communities grow richer, their concerns in depreciating assets grow bigger. The speeding up of technology makes for shorter asset lives and therefore still higher depreciation costs. In the expensively equipped factories of the future, one may reasonably expect depreciation to be a big – perhaps the biggest – element in the costs of production and service.

Regulated industries determine their allowed revenues from the revenue requirement equation and it can be defined by the following equation:

$$R.R. = O.E. + T + D + (V - d) \times ROR \qquad (1)$$

Where R.R. is the revenue requirement, O.E. is the operating expense less depreciation, T is the taxes, D is the depreciation cost, V is the gross valuation of property, d is the accrued depreciation, ROR is the rate of return on the rate base (V – d) and (V – d)xROR is the earnings allowed on the rate base. Depreciation has a direct impact on regulated rates and an indirect influence on revenues. It affects revenues as a current operating cost and the accrued depreciation cost is subtracted from the value of property in determining the rate base.

In early 1984, the Bell Operating Companies (BOCs) divested from American Telephone & Telegraph Company (AT & T). This divestiture put a stop to the monopolistic control AT & T had over the telephone industry, but also created more concern over the depreciation reserve

deficit in the BOCs. Just before divestiture, AT & T reported its

estimated depreciation reserve deficiency to be over $26 billion and

growing at a rate of $2 billion per year [1]. A separate study by the

United States Telephone Association (USTA) capital recovery committee

indicates that the telephone industry will only achieve approximately

29% of reserve level by year 1990. Based on an expected investment

base of nearly $235 billion by year-end 1990, this would indicate a

potential deficiency of nearly $40 billion [2]. Depreciation cost

alone accounts for a large percentage of the industry's revenue

requirement and in some companies, for example, Illinois Bell Telephone

company, depreciation cost is the single largest cost.[1]

Once the importance of the depreciation estimate is determined, it

is desirable to understand the process upon which these estimates are

based. The processes which depreciation estimates are based can be

generally classified into two distinct procedures: life analysis and

life estimation. Life analysis is the process of aggregating and

analyzing the historical record of property for purpose of obtaining

information about the historical patterns of the retirements of assets.

Life estimation is based on the results of the life analysis, but

proceeds one step further by applying considered judgement to the

results of the life analysis. This represents the application of

subjective "expert opinion and judgement" to estimate the mortality

---

[1]Letter from T. L. Cox (v.p. Finance, Illinois Bell) to W. J.
Tricarico (secretary, FCC), Ref.: 1984 Represcription of depreciation
rates for Illinois Bell, July 20, 1984.

characteristics of the property.

The traditionally used models of life analysis are Iowa type curves, Gompertz-Makeham formula, h-curves, simulated plant records, computed mortality, etc. The majority of these models derive service lives as a result of estimating the relation of age to the retirements of assets. The causes of retirement, often referred to as forces of mortality, are broadly categorized as [3]:

1. physical conditions (wear and tear from use)

2. functional situations (technological obsolescence)

3. situations unrelated to the property (management policy)

Of these causes for retirement of property, physical conditions were playing a major role in the retirement of plant in the past. During the last few decades, technological obsolescence and competitive factors are judged to be dominant forces of mortality. Traditional life analysis techniques would just work on the retirement data without specifically classifying the causes of retirement.

Fitch and Wolf [4] identified the need to examine individual forces of mortality and conceptualized on how those forces could be combined to give better life forecasts. They introduced the concept of statistically combing forces affecting the retirement of property under two conditions:

1. the variation in forces caused by the combination of significantly different physical characteristics or property located in distinctly different environments,

2. the forces of retirement acting in combination, but
   independently, e.g., deterioration, inadequacy, obsolescence
   and chance.

This technique may be used to estimate the effect of each force of
mortality on significant vintages when evaluating life analysis or
modifying them when making life estimation.

Wolf [5] in particular supported Ocker [6] in the belief that
technological obsolescence is the key element causing change in force
of mortality dominating all others in telecommunications industry and
should thus be studied separated. He identified three steps necessary
to obtain a life forecast when a particular force is disaggregated.
For technological obsolescence, he suggested:

1. the estimation of the effect of all forces except
   technological obsolescence,

2. forecast of the future rate of technological obsolescence,

3. the combination of these forces of mortality to yield a
   service life forecast.

He pointed out that the most critical of the three steps was
accomplished by reconstructing the life cycles of various technologies.

White [7] discussed economic forces of retirement and presented
examples showing how knowledge of those forces can be used to improve
life forecasting. These economic forces alter the short and long range
effect of the usual forces of mortality. He described how individual
forces of retirement affect the overall retirement pattern. If

unrecognized, these forces may cause historical analyses to be misinterpreted when making life forecasts.

Dandekar [8] argued that the concepts of the retirement rate being a function of age, as used in some traditional life analysis techniques, might have to be augmented to a more extended concept that not only relates the retirement rate to age but also to chronological time. He presented an overview of the product life cycle concepts and their use in the area of capital recovery. A generalized model for the investment life cycle is developed and tested against data from five different technologies. From these models, standard life cycle curves have been generalized.

Kateregga [9] discussed the need for technological forecasting as an input to life studies with the telecommunication industries as a particular case study. It was recommended that the normal and the Gompertz growth models be considered along with the Fisher-Pry model by those industries presently considering the implementation of substitution analysis in their life estimations. It was suggested that when substitution analysis is used in life estimation, the life cycle forecasts derived from the analysis be used as an additional constraint to future experience before service life indications are derived.

Fitch, Wolf and Bissinger [10] stated in there preface to "The Estimation of Depreciation" that:

> The effect of advances in technology, technological forecasting, life cycle costing and life cycle depreciation are current topics related to depreciation which need to be incorporated in life estimation studies.

This new dimension of life analysis will, more appropriately, account for the technological and obsolescence factors leading to retirement. In a technology driven environment, a depreciation estimate which is based on traditional life analysis results in a decelerated rate of capital recovery. This time pattern of technological growths models needs to be incorporated into the life analysis framework especially in those industries experiencing fast technological changes. Technological forecasting in life analysis is based on the premise that, if one can forecast not only the onset of data, but also the pattern of development of particular technologies, then one can estimate better the lives of affected equipment.

CHAPTER II. TECHNOLOGICAL FORECASTING METHODS

Technological forecasting methods are tools which are used for planning and decision making in order to obtain insight on the future of a technology, group of technologies or undiscovered technologies, and their direction of change and advance over the longer time. There are various definitions of technological forecasting. One of the earliest pioneers, Lenz [11], has described it as follows:

> Technological forecasting may be defined as the prediction of the invention, characteristics, dimensions, or performance of a machine serving some useful purpose. The qualities sought for the methods of prediction are explicitness, quantitative expression, reproducibility of results, and derivation on a logical basis.

The definition given by Bright [12] is as follows:

> Technological forecasting is a quantified prediction of timing and of the character or the degree of change of technical parameters and attributes associated with the design, production, and use of devices, materials, and processes, according to a specified system of reasoning.

Jantsch [13] defined the technological forecasting as the probabilistic assessment on a relatively high confidence level of future technology transfer. Landford [14] defined the technological forecasting as the prediction or determination of the feasible to desirable characteristics of performance parameters in future technologies. A widely accepted definition of technological forecasting has not been formulated to date. One reason that a single definition has been elusive is that various researchers see different meanings in different disciplines. As a result, some of the terms

associated with the technological forecasting might not have consistent meaning across various areas. The term of technological forecasting within life analysis has come to mean the forecasting of process from birth to death of a product. Methods of technological forecasting have historically been broken down into three categories: subjective assessment, exploratory, and normative methods [13, 15, 16, 17, 18].

Subjective Assessment Methods

Subjective assessment methods are not generally based on quantitative inputs. This group of methods depends largely on one or more experts who are knowledgeable in the specific technology or technical area under consideration to assess the future.

Low effort intuitive

This group of technological forecasting methods include techniques which make use of the knowledge of experts without using any elaborate framework to obtain the experts' forecasts. Examples of these methods are brainstorming sessions, visionary forecasts, or simply asking individuals who are experts in their field what they expect to happen in the future. Certron and Monohan [19] and Markridakis et al. [16] described considerable merits in a forecast made by a single individual who is expert in his special area.

The best known technique is brainstorming and is well described by Ayres [20]. This method consists of group meetings conducted under a

set of simple rules designed to created an environment conductive to
freewheeling speculation, such as the following:

- focus on a single well-defined problem, but

- consider any idea, regardless of apparent relevance or
  feasibility,

- do not criticize any idea,

- do not explore the implications of idea.

Chambers [21] explained another typical low effort intuitive
forecasting method called visionary forecasts:

> A prophecy that uses personal insights, judgement and, when
> possible, facts about different scenarios of future. It is
> characterized by subjective guesswork and imagination; in
> general, the methods used are non-scientific. It requires a
> set of possible scenarios about the future prepared by a few
> experts in light of past events.

One problem with the use of expert judgement is determining who
are real experts. According to Helmer [22] there are three basic rules
which should be followed when dealing with expert judgement:

- the experts must be selected wisely,

- the proper conditions under which they can perform most ably
  must be created,

- if several experts forecast a particular subject, considerable
  caution must be used in deriving a single combined position.

The primary advantages of low effort intuitive forecasts are that
they are simple, usually cheap to produce, and frequently they can be
made quickly and easily. The main disadvantage is their heavy reliance
on intuitive opinions and the lack of quantitative data. In any case,

it is usually best to start with a low effort intuitive forecast and then use the more complex and costly technological forecasting methods when needed.

High effort intuitive

This group of technological forecasting methods includes techniques which make use of formal frameworks. Examples of these methods include polls and panels, Delphi technique, and cross impact analysis, each of which provide a framework for obtaining, combining and presenting the combined forecasts which are obtained from multiple experts. Chambers [21] discussed high effort intuitive techniques stating that:

> The objective here is to bring together in a logical, unbiased, and systematic way all information and judgements which relate to the factors being estimated. Such techniques are frequently used in new technology areas, where development of a product idea may require several inventions, so that R & D demands are difficult to estimate, and where market acceptance and penetration rates are highly uncertain.

Ayres [20] clearly explained the reason why higher effort intuitive forecasting methods are needed:

> If a certified forecasting genius is available, of course, there is nothing more needed. If, however, either the genius of the certification is in question - which is normal - the modern tendency is to rely instead on an opinion poll or a panel of experts.

The use of panels or polls is an attempt to use simple averages to obtain a reliable forecast. Frequently the poll or panel uses interactions among the participants in order to obtain a group

consensus. One of the best known methods to develop a group consensus
is the Delphi technique which is explained very concisely by Helmer and
Rescher [23]:

> The Delphi technique eliminates committee activity altogether
> thus further reducing the influence of certain psychological
> factors, such as specious persuasion, the unwillingness to
> abandon publicly expressed opinions, and the bandwagon effect
> of majority opinion. This technique replaces direct debate
> by a carefully designed program of sequential individual
> interrogations (best conducted by questionnaires),
> interspersed with information and opinion feedback derived by
> computer consensus from the earlier parts of the program.

This technique involves the use of a questionnaire asking selected

experts in many fields the approximate time period they expect the

specified events to occur. A systematic solicitation of expert opinion

works toward a consensus by the use of sequential interrogations.

Following receipt of the responses to the first survey, the individual

or organization conducting the survey prepares a second questionnaire

providing results of the first survey, and asking individual who

provide responses outside the interquartile range (the middle fifty per

cent of response) to give reasons why their answers depart from the

response obtained in the first round. The third round of the survey

provides all participants with the justification or reasoning behind

the extreme answers - i.e., those answers at either end of the

distribution for the time frame being considered. The fourth round

gives all participants an opportunity to consider the reasons given for

extreme positions and participants are asked for one last opinion.

The cross-impact analysis was devised by Gordon [24] and Helmer [25], subsequent to their original Delphi study. It arose from an objection to the Delphi technique, that the experts were expected to concentrate on one event at a time, and might not be able to take into account the interactions among the events they were forecasting. This method requires determination of the potential impact of one of the forecasted events on each of the other forecasted event. The following set of steps for this analysis are modifications of those originally outlined by Rochberg et al. [26]:

1. Select one of the forecasted events to be compared and subjectively determine the original probability of occurrence.

2. Assuming the occurrence of the first event, assign probabilities of occurrence to the other forecasted events, taking into account the enhancement or inhibition effects of the first event.

3. Perform a similar exercise on each event, with the assumption that the presently being considered event is now the first event and the probabilities of each of the other events determined should this first event (each in its turn) occur.

4. Continuing this process until all events in the set have been decided.

5. By computer application, compute the matrix many times to compute probabilities based on the number of times or percentage of times that an event occurs during these repeated exercises.

6. Changing the initial probability of one or more events and repeating steps 2 through 5.

Through application of these procedures, an analyst can develop a set of probabilities that adequately represents the interaction between a number of different events, each of which is uncertain.

Exploratory Methods

Exploratory methods begin with the past and present as their starting point and project the future in a heuristic manner, often looking at all available possibilities.

Trend extrapolation methods

This group of technological forecasting methods includes various techniques which make use of historical data in order to construct a trend which is then extended or extrapolated into the future. Trend extrapolation methods assume that:

1. those forces which created the prior pattern of progress will be more likely to continue than to change,

2. those forces are more likely to extend the previous pattern of progress than to produce a different pattern.

The more important techniques for trend extrapolation methods are graphical trend extrapolation, curve fitting, substitution models, envelope curves, and analogy methods. The shape of the trend curves can be taken any one of various curves including the exponential growth curve, the life cycle curve which is an S-curve which has leveled off, or straight line growth curves.

Curve fitting is a methodology commonly used as a quantitative forecasting technique, but it is also an approach that can be effectively used as a technological forecasting method. Many different shapes of curves can be used in fitting historical data for technological forecasting. One of the most frequently used forms is that of the S-curve. This curve implies a slow start, a steep growth, and then a plateau that is characteristic of many technological capabilities. This shape of curve is often used in depicting the product life cycle. It is a difficult problem to determine which form of curve will best fit the available data and give an accurate forecasting for the future.

Substitution models show graphically or mathematically how one technology replaces another (or multiple) competitive technologies. This method assumes that if one product or technology exhibits a relative superior in performance over the established or conventional product or technology, it will eventually substitute for the product or technology of lesser performance. An excellent description of using substitution models can be found in Linstone and Sahal [27].

Envelope curves are constructed by combining the individual trend
curves of various competing technologies over time. Ayres [20]
described the rational of the envelope curves as:

> The justification for this notion, in brief, is that the past
> performance of the system is fairly likely to be a good model
> for its future performance; i.e., the system stimulates
> itself. One would expect this to be true, as long as, and to
> the extend that, there is no radical change in the
> environment. In this light, it is plausible to expect the
> amount of creative work - which is equivalent to, or closely
> related to, the rate of invention - from year to year to
> increase (or decrease) relatively steadily and gradually,
> rather than fluctuating sharply up and down. This
> characteristic rate of change is, of course, the slope of the
> envelope.

The envelope curve then represents a cumulative curve which can then be
extrapolated to show where potential future technologies may appear in
the future.

Analogy methods attempt to compare historical patterns with
existing situations in order to forecast future progress. The concept
of an analogy between biological and other kinds of growths has
recently been applied to such phenomena as the growth pattern of
particular technologies, transportation speeds, the life cycle of
individual products, and the growth of government spending [28]. When
data points are plotted and the curve follows the S shaped pattern,
forecasters may choose to use a relationship or formula developed for
use in some other discipline for predicting future data points.

Trend correlation methods

This group of technological forecasting methods includes the various techniques which make use of correlations of trends in order to forecast a variable based upon the behavior of other related variables. In order to use two or more trends to determine a third, the forecaster must have available a number of primary trends which are related to the technical field of interest. To these he must add a knowledge of probable relationships that might arise from combinations of such variables. The forecaster may then select the relationship and the primary variables which influence the desired technical improvement. The trends of the primary variables may be projected on the basis of any techniques which appear appropriate. The prediction is then completed by projecting the unknown variable on the basis of the relationship between the primary variables. Examples of these techniques include precursor events, correlation analysis, and econometric models.

Precursor events and leading indicators are used when a relationship is known or believed to exist between the variable being predicted and the leading variables. By following the leading variable or precursor event, the variable can be forecasted based upon the leading-lagging relationship. In the past, it was believed that only technical progress could serve as a precursor. Research indicated that economic, political, and social factors may also serve as precursors. Although it is difficult to determine precursive indicators, the

discovery of one adds to the confidence of the forecaster in his overall results.

Correlations, regressions and multiple regressions models make use of a known or believed casual relationship between the variables. Once the correlations or regression models are determined, the variable can be forecasted based upon the behavior of related variables. It is most desirable that all factors which have a bearing on the outcome be considered, and that any relationships among them which are known a priori be defined in advance.

Econometric models are usually large, complex regressions models which can often be used to provide information on the general economy. This information can then be used or correlated with the variable being predicted in order to forecast the future of the variable. Chambers [21] explained as:

> Econometric models are a system of interdependent regression equations that describe some sector of the economic sales or profit activity. The parameters of the regression equations are usually estimated simultaneously...due to the system of equations inherent in such models, they will better express the causalities involved than an ordinary regression equation and hence will predict turning points more accurately.

Frequently in technological forecasting, it is desired to forecast a technology or a progress without a long or any history and often causal relationships are not known. This makes the application of trend correlation methods nearly impossible and requires the use of other methods.

Normative Methods

This group of methods includes various models which are used to explore what the future potential success of a technology or a goal will be given the various assumptions used to construct the models. These models also allow experimentation with the assumptions in order to test their affects on the potential success of model. The more important models for normative methods are scenarios, gaming, relevance trees, morphological analysis, and simulations.

Scenarios take a well defined set of assumptions, then develop the process of constructing multiple future situations, the paths to get to each of these situations, and the comparative analysis of the success of the particular technology being forecasted relative to other competing technologies. Each scenario consists of a picture of a plausible future situation including all relevant environmental assumptions. These alternative scenarios are described by Bright [29] as:

> The alternative scenario concept is to construct a number of possible futures and to plan future action only after considering all these scenario that represent possible futures. The forecaster or planner does not insist on identifying the most probable events but uses scenarios that include significant differences. Scenarios, therefore, should be developed to span a wide range of changes, not only in technological areas but also in economic, political, social, and other conditions.

Bright [29] added that forecaster should develop plans under each alternative scenario and then compare the separate plans to see where they are similar and where they are different. Then the final plan

developed should be tested in each scenario to see how well the plan

survives each scenario. Multiple scenarios (often three) are

frequently used as the base assumptions in other technological

forecasting, i.e., Delphi, trend analysis, and simulation models. Much

of works on scenarios have been developed by Kahn [30].

A technique that is very similar to that of scenarios in concept

is gaming or role playing. Ayres [20] described the technological

forecasting method of gaming:

> ...in which each participant is asked to play a specific
> role in a scenario. The initial conditions only may be
> specified in advance, or an outline of the further action may
> be provided. The participants may be allowed to play their
> parts quite freely, or under severe constraints (the rules of
> the game), depending on the purpose of the exercise.

A refinement of the scenarios is the relevance tree. The

principle in relevance tree analysis is to define an objective and

hierarchy of tasks, systems, and technologies that need upgrading to

achieve that objective. They are developed by evaluating the relative

importance of each element of this tree against explicit, quantified

criteria and a specific scenario, at progressively lower levels.

Morphological analyses are used when the system or process can be

broken down into parts which can be treated independently with several

solutions or approaches to each part and where there are no

hierarchical relationships. Zwicky and Wilson [31] described five

basic steps that constitute the morphological analysis as:

1. The problem must be explicitly formulated and defined.

2. All parameters that may enter into the solution must be identified and characterized.

3. A multidimensional matrix (the morphological box) containing all parameters identified in step 2 must be constructed. This matrix will contain all possible solutions (combinations).

4. All solutions of the morphological box should be examined for their feasibility, and analyzed and evaluated with respect to the purposes which are to be achieved.

5. The best solutions identified in step 4 should be analyzed (possibly through an additional morphological study) as to the feasibility of carrying them out with available resources.

Like relevance trees, morphological analyses involve a large number of alternatives, all of which must be assessed and evaluated. This process requires considerable human and computer resources.

Simulation models are primarily mathematical models which attempt to imitate the future environments of the technology in order to investigate, experiment with, and compare the success of the technologies under study. The use of simulations in industry started with Forrester [32]. His approach was initially called the Industrial Dynamic model and has evolved into very useful methods with which to study a technology or group of technologies. The goals of this method are:

- To develop a better understanding of time-varying behavior.

- To show interrelationships among the major aspects of a system.

- To help predict the future course of an existing system.

- To help improve the prospects for the future.

Simulations models are often used to predict future probabilities under various assumptions. They require an explicit model structure and usually operate with very dynamic interactions in the process leading to the forecasts.

CHAPTER III. OBJECTIVES OF STUDY

The subject of this study is the application of the technological forecasting methods to the process of life analysis. It has been observed that a similarity exists between the behavior of biological growth and the pattern of technological growth that has some upper limit. Some analysts prefer to employ a particular form of the growth model for all technological growth patterns and other analysts may prefer to employ mathematical transformations of these models. But no method has been developed to determine the best model.

The major objectives of this study are as the following:

1. To study technological growth models and to develop a technique that allows the analyst to select the best model as technological forecasting method for the data under examination.

2. To incorporate the time pattern of technological growth into the life analysis framework, the more recent data points should have a greater influence on parameter estimation than the data that occur at the early part of the growth model.

3. To develop a methodology to find the remaining life and the required accumulated depreciation of embedded plant based on technological growth models.

CHAPTER IV. OVERVIEW OF GROWTH MODELS

The technological forecasting methods are frequently grouped to explain and predict the future growth in functional capability of some specific technology. Many trend extrapolation methods and techniques are being developed to forecast technological growth. However, only limited attempts have been made to apply quantitative methods to real life data patterns. One of the quantitative forecasting techniques in the realm of technological forecasting methods is curve fitting utilizing the growth models. The term growth model refers to a plot over time of some capability or characteristic of a particular technology. It is generally believed that this curve, plotted on arithmetic scales, reflects a slow start followed by exponential growth, and then levels off against some upper limit produced by nature or technical capabilities. This pattern of growth results in, what is termed, an S shaped curve with respect to time.

Several qualitative attempts have been made to justify the analogy between technological growth and biological growth with some form of an upper limit. In particular, Lenz and Landford [33], who have performed much research in the realm of technological growth curves, linked the biological growth to technological growth and arrived at the same formulations. In addition to the simple visual recognition of an S shaped growth curve, several researchers [12, 34] have tried to justify the growth pattern in technology.

- "Slow initial growth" The new technology emerges slowly at the onset partially due to little involvement by too few people, minimal preliminary scientific knowledge, and engineering obstacles which must be overcome. There may be a lack of scientific and technical understanding, and it may take time to overcome conventional wisdom and erroneous assumptions about the phenomena involved. Contract arrangements for the old technology are still in force and can not be violated. There are also production diseconomies due to small scale and problems of financing, developing, and installing. Consumers postpone acceptance in anticipation of changes in quality and price and there is a low elasticity of supply of the new technology. Lakhani [35] discussed the effect of the age distribution of existing capital stock on the rate of growth during this stage. Since new processes usually require new capital equipment, firms with relatively old equipment would be prompt in accepting the new technology. The speed of take-off the new technology will then be determined by that age distribution. Mansfield [36] studied the effect of the age distribution relationship in the railroad industry and found that the older the steam locomotives (i.e., old technology) of the firm, the faster the diesel locomotives (i.e., new technology) were adopted.

- "Exponential growth" Once some of these preliminary obstacles
  have been overcome, the growth of the technology begins to
  follow an exponential pattern. Once understanding and proof
  of the technology's capabilities have been attained, funds and
  researchers are made available to the technology. The product
  has become accepted and production processes improved.
  Economies of scale have set in with consequential reduction in
  prices. New contracts have been made and the learning process
  ended. The quality of the process has surpassed that of the
  older technology and the information gap between producers and
  consumers has been bridged. The bandwagon effect has begun.
  The efforts are applied not only to the pure research behind
  the technology, but also to the refinement and employment of
  that technology in an engineering environment.

- "Leveling off" Finally, the technological advances cease to
  accelerate and even to grow. Growth itself may halt due to
  the exhaustion of technical opportunities for further
  advances, the point of diminishing returns in cost, effort,
  and usefulness may have been met, or perhaps the physical
  limits of the technology itself may cause the asymptotic
  behavior of the growth curve. In this latter stage of
  development, the product has essentially exploited its scale
  economies, it has matured and is no longer changing rapidly.
  But probably the most critical limitation is the virtual

circulation of the available market. A new product could be

introduced as a substitute at this time, not only forcing the

growth to stop, but also initiating a decline phase.

Various growth models have been proposed to represent the time pattern

of technological growths. For practical purpose, these curves

classified by the degree of skewness: symmetric growth models and non-

symmetric growth models.

Symmetric Growth Models

The Pearl growth curve

Some researchers observed a similarity between the pattern of

biological growth and the growth in the performance capability of a

particular technology. The original work done on the biological growth

curve was performed as early as 1838 by a Belgian mathematician, P. F.

Verhulst [37]. Some species of animals and some bacterial cultures

were observed to grow rapidly at first when placed in a limited

environment with ideal conditions of food supply and space for their

initially few numbers, and then to grow slowly as the population

approaches a point where there was pressure on available resources. It

was rediscovered and popularized by Pearl and Reed [38] about 1920.

Pearl [37] made extensive studies on the growth behavior of organisms,

e.g., the rate of increase of fruit flies within a bottle, the rate of

increase of yeast cells in a given environment, and the rate of cell

increase in white rats. He described the mathematical function of his

results by what has become known as the Pearl growth curve or the logistic curve.

The equation of this curve is,

$$Y = \frac{L}{1 + \alpha e^{-\beta t}} \qquad (2)$$

where

Y = the technological variable being achieved at time t,

L = the upper limit to that technological capability,

t = the value of time,

α, β (α, $\beta > 0$) = the parameters of the model.

In this equation, α and β are parameters which control the shape of the growth curve. The value of β determines the steepness of the exponential growth portion, while the value of the α determines the position of the curve on the time (t) axis. The curve has an initial values of zero at time $t = -\infty$, and reaches the limiting value L at time $t = +\infty$. By setting the second derivative of Y with respect to time equal to zero, it can be shown that the inflection point of the curve occurs at $t = (\ln \alpha)/\beta$, where $Y = L/2$. The curve is symmetric with respect to the inflection point and is presented in Figure 1.

The Fisher-Pry model

One of the well-known growth models available in technological forecasting models is the Fisher-Pry model, which is sometimes used in substitution analysis. Fisher and Pry [39] explained a technology as a
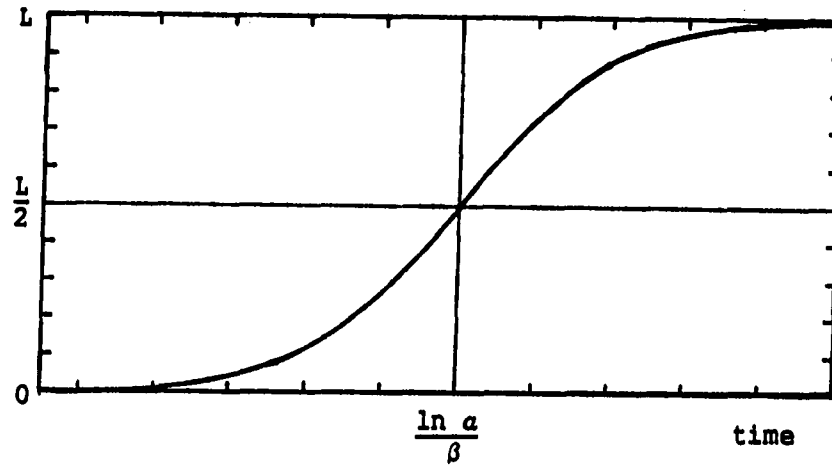
FIGURE 1. Pearl growth curve

set of substitution processes and showed that substitutions tend to proceed exponentially in the early years, and to follow an S shaped curve. The model based on three assumptions:

1. many technological advances can be considered as competitive substitutions of one method of satisfying a need for another,

2. if a substitution has progressed as far as a few percent of the total consumption, it will proceed to completion,

3. the fractional rate of fractional substitution of new for old is proportional to the remaining amount of the old left to be substituted.

They created a model in which the substitution proceeds at a rate determined by the formula,

$$Y = \frac{1}{2} [1 + \tanh a(t - t_0)] \qquad (3)$$

where

Y = fraction of growth of the technology,

tanh = hyperbolic tangent function,

a = half of the annual fractional growth in the early years,

t_0 = the time in which the new technology captures 50

percent of the usage or Y =1/2.

A more convenient form of the above substitution expression is,

$$\frac{Y}{1 - Y} = e^{2a(t - t_0)} \qquad (4)$$

The mathematical derivation of this model is given in Appendix A.

Such a model is appropriate for advancing technology which exhibits a relative improvement in performance over the older technology. The appropriate measure of the technology is the percentage of take-over. Lenz and Landford [33] demonstrated that predictions of take-over may be made when a technology showing superior performance has substituted for as little as 2 % of the total market. This model can be derived from the Pearl growth curve, but there exist some differences in the fit due to the estimation procedure. The algebraical derivation is provided in Appendix B. In the case of the

Fisher-Pry model, the dependent variable is the ratio of $Y/(1 - Y)$. After this form of the model has fitted, it is transformed to give Y. Since the Pearl growth curve fits Y directly, the two resultant curves are not identical.

The Mansfield-Blackman model

Perhaps the seminal work in the modelling of the process by which a new product substitutes for an existing product is due to Mansfield [40]. He was able to illustrate that the growth in the number of users of an innovation can be approximated by a logistic curve by analyzing four disparate industrial sectors: railroads, coal, steel, and breweries. The key assumption behind Mansfield's model is that the probability that a firm will introduce a new technique is an increasing function of the proportion of firms already using it and the profitability of doing so, but a decreasing function of the size of the investment required.

Blackman [41] made a revision in Mansfield's model by modifying the definition of substitution. He defined the substitution process in terms of market share captured by the new technology, rather than in Mansfield's terms of the accumulative numbers of firms which employed the new technology. Blackman believed that the motivating force behind the creation of technological innovation is the economic reward which is perceived to result from successful innovations. The extent of investment in technological innovation is, therefore, related to the

perceived rate at which a market will develop for a new technology, and the rate of market development is a function of technological substitution. He applied his model to describe the dynamics of innovation in the commercial jet engine market [42] and in the electric utility and automotive sectors [43].

The Mansfield-Blackman model is,

$$\ln \left(\frac{Y}{L - Y}\right) = \ln \left(\frac{Y_0}{L - Y_0}\right) + a(t - t_1) \tag{5}$$

where

Y = market share captured at time t by new innovations,

L = upper limit of market share which the new innovation
    can capture in the long run,

Y_0 = market share captured when $t = t_1$,

a = constant which governs substitution rate,

t_1 = the time in which the innovation first captures
    a portion of the market.

If $\beta_0 \equiv \ln [Y_0/(L - Y_0)] - at_1$ and $\beta_1 \equiv a$, then equation (5) can be rewritten as,

$$\ln \left(\frac{Y}{L - Y}\right) = \beta_0 + \beta_1 t \tag{6}$$

Note that the Mansfield-Blackman model is simply the Fisher-Pry model when the upper limit L is set to 1. For this reason, it will not be treated as a separate model.

The Bass model

Bass [44] developed a somewhat different approach to the introduction of new products concerned with the development of a theory explaining the timing of a the initial purchase of new products for consumer durables. He pointed out that Haines [45], Faurt and Woodlock [46], and others have suggested growth models for new brands or new products which assumed exponential growth to some asymptote. Bass, however, postulated a growth of sales to some peak and then a leveling off of sales at some level less than the peak, indicating that replacement sales began to dominate.

Two key assumptions behind this model of purchasing behavior are:

- Initial purchases of the product (or technology) are made by both innovators and imitators. Innovators are not influenced in the timing of their initial purchase by the number of people who have already bought the product, while imitators are influenced by the number of previous buyers.

- The importance of innovators will be greater at first but will diminish monotonically over time.

Bass proposed the growth model to utilize the derivative dy/dt, rather than deal with the cumulative form of the model:

$$S(t) = pm + (q - p)Y(t) - \frac{q}{m}[Y(t)]^2 \tag{7}$$

where

$S(t)$ = the predicted sales at time t,

Y(t) = the cumulative sales of consumer durables in time

interval (0,t),

m = the market potential for first time purchasers,

p = the coefficient of innovation,

q = the coefficient of imitation to reflect the word-of-

mouth communication between adopters.

This model is based on the Pearl growth curve and the derivation of the
Bass model from the logistic function is given in Appendix C.

Nevers [47] tested the applicability of Bass's model to four
different sectors of the economy: retail service, industrial durables,
agricultural durables, and consumer durables. He found that the model
generated rather accurate predictions of the general time path of
product and technological adoptions in these sectors based on the role
of innovators and imitators in adoptive behavior. Heeler and Hustad
[48] reported a dramatic improvement in the quality of forecasting by
constraining the value of m (i.e, the market potential for first time
purchasers) to an intuitive estimate. The methodology used by those
authors was based on a modification of the routine developed by Eide
and Ness [49], which employed an iterative estimation procedure based
on prespecified levels of m.

Non-symmetric Growth Models

The Gompertz growth curve

An another growth growth curve that has been employed extensively in technological forecasting is the Gompertz curve [50], named after Benjamin Gompertz, who originally proposed the Gompertz curve as a law governing mortality rates. The mathematical form of this curve is,

$$Y = L \cdot e^{-G \cdot e^{-kt}} \tag{8}$$

where

Y = the technological variable being achieved at time t,

L = the upper limit to that technological capability,

t = the value of time,

G, k (G, k > 0) = the parameters of the model.

Similar to the Pearl growth curve, the Gompertz growth curve ranges from zero at $t = -\infty$ to L at $t = +\infty$. By setting the second derivative of Y with respect to time equal to zero, in contrast to the Pearl growth curve whose inflection point occurs at L/2, the point of inflection occurs at t = (ln G)/k, where Y = L/e. The curve is not symmetric with respect to the inflection point and can be seen in Figure 2.
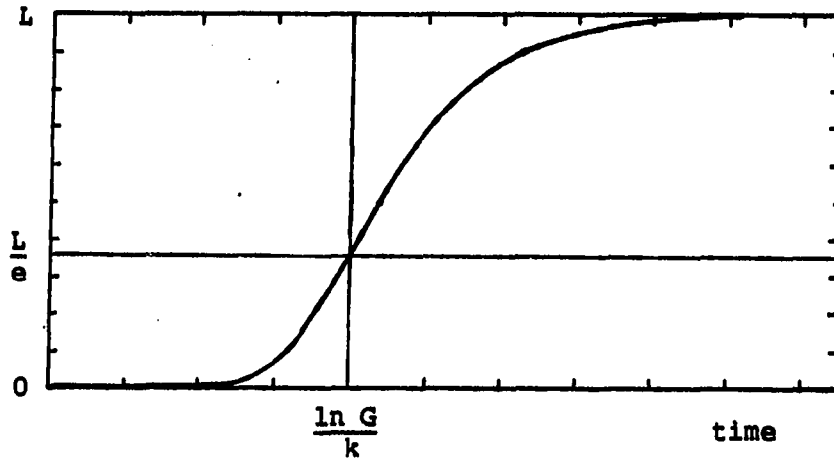
FIGURE 2. Gompertz growth curve

Floyd model

Floyd [51] wished to develop a means of analyzing historical
technological growth and to forecast growth trends in improved
functional capability (i.e., the ability of technology to carry out its
function). This capability is translated in "figures of merit" which
serve as the dependent variables (e.g., thrust to weight and thrust to
rate of fuel consumption for aircraft engines, efficiency of energy
conversion, or inventory requirements). His "figures of merit"
analysis is based on three requirements:

- an ultimate limit can be calculated or estimated,
- the competitive technology limit can be estimated, and

- at least two data points on figures of merit and time be
  available.

Floyd assumed that there is a fixed number of techniques that can be tried to accomplish a specific goal and that only a fraction of these will be successful. As implied in the requirements, there is an upper limit to the functional capability which will be reached only after all the possible techniques have been tried. His model can be summarized as,

$$P(Y, t) = 1 - \exp[\frac{-0.6931(c_1 t + c_2)}{F + \ln(F - 1) + c_2}] \tag{9}$$

where

$P(Y, t)$ = probability of achieving figure of merit level Y

by time t,

L = limiting value of figure of merit,

Y = level of figure of merit for new technology,

Y_c = level of figure of merit for competitive

technology,

c_1, c_2 = constants,

t = time,

$F = (1 - Y_c/L)/(1 - Y/L)$.

Sharif and Uddin [52] developed a procedure for adapting available mathematical models for forecasting technological substitution and revised the Floyd model as,

$$\ln (\frac{Y}{L - Y}) + \frac{L}{L - Y} = c_1 + c_2 t \tag{10}$$

where

      L = upper limit of the market share,

      Y = market share of a substitute product at time t,

      c_1, c_2 = constants.

In this form, it can be seen that the Floyd model is a modification of the Mansfield-Blackman model. The term $L/(L - Y)$ has been inserted to allow for a time-decreasing coefficient of delay, whereas the Mansfield-Blackman assumes that this coefficient of imitation is time-invariant. Sharif and Uddin suggested the following procedures:

1. Use Blackman's model to forecast the market share if the value of maximum market share can be reasonably ascertained. Otherwise go to step 2. If Blackman's model is used, plot a set of curve by taking a range of values instead of just one value for the anticipated maximum possible market share. Go to step 4.

2. Use Fisher's model and Floyd's model (with $L = 1.0$) to obtain a range of feasible forecasts.

3. Plot a set of curves, within the region obtained in step 2, using Fisher's curve as the stem and branching off at different points with decreased values of the rate of growth of substitution.

4. Identify the most likely forecast based on judgment with due consideration to the environmental factors that may influence the rate of growth of substitution to change over time.

The Sharif and Kabir model

Sharif and Kabir [53] developed a generalized mathematical model for forecasting technological substitution under a wide variety of circumstances. The Floyd model gives an underestimation of the forecast, while the Mansfield-Blackman and Fisher-Pry models give an overestimation of the forecast [52]. Since it is likely that the correct estimation lies between these two extremes, Sharif and Kabir suggested that the models of Floyd and of Mansfield-Blackman be linearly combined, as follow:

$$\ln\left(\frac{Y}{L - Y}\right) + \sigma\left(\frac{L}{L - Y}\right) = c_1 + c_2 t \tag{11}$$

when

$\sigma = 1$, the Floyd model results,

$\sigma = 0$, the Mansfield-Blackman model results,

$\sigma = 0$ and $L = 1$, the Fisher-Pry model is evoked.

The term $L/(L - Y)$ is labelled a delay factor and σ is termed the delay coefficient. Since σ can be take a value between zero and one, a set of smoothed S shaped curves can be obtained, ranging from the most optimistic to the most pessimistic forecast.

The Weibull growth curve

The Weibull distribution bas been found experimentally to describe industrial property mortality characteristics, such as, vacuum-tube failures [54] and ball-bearing failures [55]. This distribution is

named after its conceiver, Waloddi Weibull [56], who derived it in an
analysis of the yield strength of a Bofors steel, the length of
Cyrtoideae, the fatigue life of a St-37 steel and others. To a lesser
extent, the Weibull distribution has been used to study biological
phenomena, such as, the response to stress. For example, Peto et al.
[57] describe "age specific cancer induction rate" with a Weibull
distribution. Sharif and Islam [58] proposed the empirical Weibull
growth curve as a general model for technological forecasting as
follow:

$$Y = L - L \cdot e^{-(\frac{t - \mu}{a})^\beta} \tag{12}$$

where

μ = a threshold or shift parameter,

a ($a > 0$) = a scale parameter,

β ($\beta > 0$) = a shape parameter,

L = a upper limit.

For technological growth cases, the upper limit L is taken to be any
desired value less than or equal to unity. The Weibull probability
density function (p.d.f.) and its cumulative distribution function
(c.d.f.) for different values of a and β are shown in Figure 3 and
Figure 4.

It can be seen that a and β together determine the steepness of
the curve, while β alone determines the shape of the curve. Changing
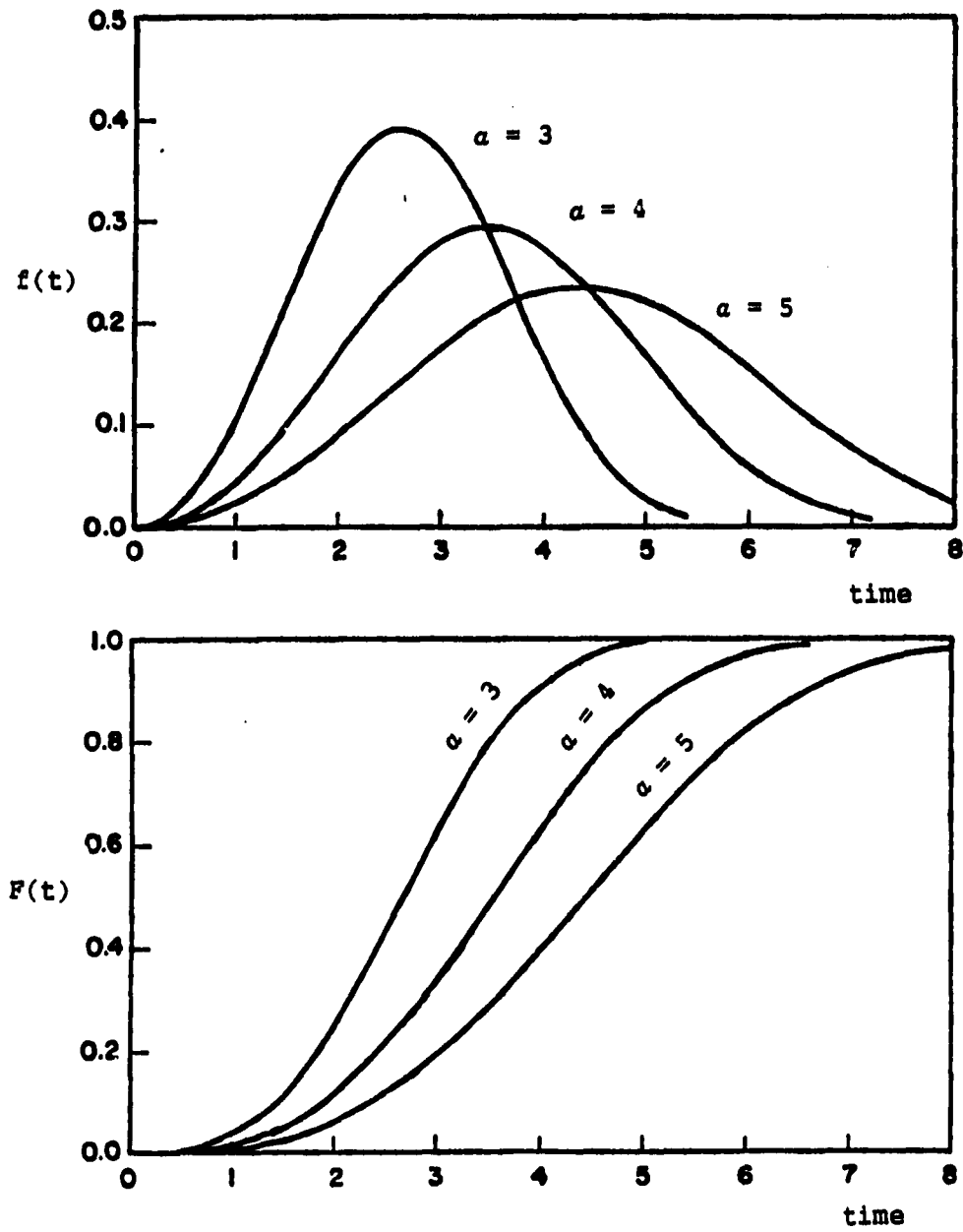
FIGURE 3. Weibull p.d.f. and its c.d.f. for different value of a when $\beta = 3$
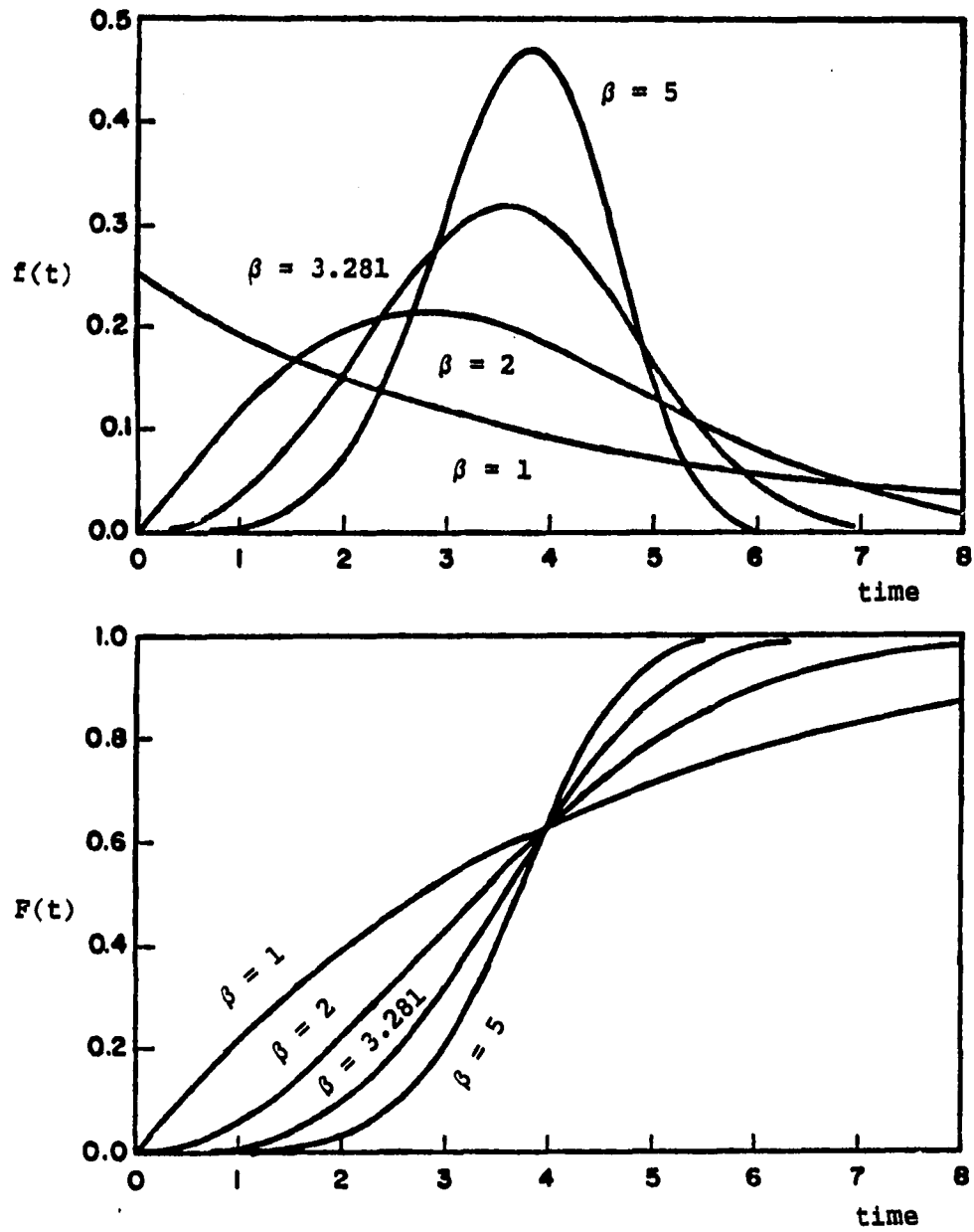
FIGURE 4. Weibull p.d.f. and its c.d.f. for different value of β when $\alpha = 4$

the value of a merely squeezes or broadens the curve, thus, just changes the scale on the horizontal axis, and not the basic shape of the curve. When β is 1, the Weibull distribution reduces to the negative exponential distribution. When β is 2, it reduces to the Rayleigh distribution and at values of $3<\beta<4$ it approximates the normal distribution. Depending on the value of β, the Weibull curve becomes left skewed, symmetrical, or right skewed. In a way, β is similar to the delay coefficient in the Sharif and Kabir model, thus, the Weibull curve effectively models technological forecasting for a wide variety situations.

In order to determine a suitable technological growth model, an overview of the growth models that have been suggested in literature. As can be observed from Table 1, the selection of a model for technological growth forecasting has an implicit relationship with the selection of the point of inflection of the S shaped curve.

TABLE 1. Growth models and their inflection points

model	expression	fraction of the upper limit when inflection occurs
Pearl	$Y = \dfrac{L}{1 + ae^{-\beta t}}$	0.5
Fisher-Pry	$\dfrac{Y}{1 - Y} = e^{2a(t-t_0)}$	0.5
Mansfield-Blackman	$\ln\left(\dfrac{Y}{L - Y}\right)=\beta_0+\beta_1 t$	0.5
Gompertz	$Y = L \cdot e^{-G \cdot e^{-kt}}$	0.368
Floyd	$\ln\left(\dfrac{Y}{L - Y}\right)+\dfrac{L}{L - Y}=c_1+c_2 t$	0.333
Sharif-Kabir	$\ln\left(\dfrac{Y}{L - Y}\right)+\sigma\left(\dfrac{L}{L - Y}\right)=c_1+c_2 t$	0.5, $\sigma=0.0$ 0.382, $\sigma=0.5$ 0.333, $\sigma=1.0$
Weibull	$Y=L-L \cdot e^{-\left(\frac{t - \mu}{a}\right)^{\beta}}$	0.5, $\beta=3.2814$ 0.333, $\beta=1.6814$ 0.368, $\beta=1.8424$ 0.632, $\beta=12.000$

CHAPTER V. ESTIMATION OF GROWTH MODELS

Choosing an appropriate growth model to represent a set of data points may be a difficult task. There exists a broad group of growth curve models as seen in the previous chapter. But the selection of the best curve is only a part of the eventual solution for modeling and forecasting the data. What also needs to be determined is the appropriate fitting procedure of the growth curve to the data.

In order to fit any curve to a set of data points, one needs an estimation procedure. One of the most commonly employed method is termed least squares estimation. This statistical procedure finds the best fitting curve for a given set of data points by minimizing the sum of the squared deviations of the observed data points from those predicted by the fitted curve. The procedure called ordinary least squares is a fitting procedure that weights all squared deviations equally for all data points. For some sets of data points, an unequal weighting procedure may be deemed appropriate. For example, if the observations are time series data, one may wish to give more weight to recent observations and discount older observations. In this way, one may guard against possible parameter changes through time.

Ordinary Least Squares

Any statistical model combines both deterministic and stochastic elements. Thus, a general form is,

$$Y_i = f(X_i, \beta, \epsilon_i), \quad i = 1,\ldots,n \tag{13}$$

where

Y_i = the dependent or response variable,

X_i = the vector of independent variable,

β = the vector of population parameters which determines

the mathematical relationship between Y_i and X_i, and

ϵ_i = the stochastic element of error.

When an additive error structure is assumed, after any new necessary transformation, the above equation can be rewritten as,

$$Y_i = f(X_i, \beta) + \epsilon_i, \quad i = 1,\ldots,n \tag{14}$$

Then $f(X_i, \beta)$ becomes the deterministic element of the model, and ϵ_i is interpreted as the stochastic element.

In practice, the vector β, is unknown and can be estimated from historical data of the variable being modelled. To perform this estimation process, the method of least squares is often used. The key concept behind this technique is the reduction of residuals (or the errors) by minimizing the sum of squares of the deviation of observed values from the estimated values,

$$S(\beta) = \sum_{i=1}^{n} [Y_i - f(X_i, \beta)]^2 \tag{15}$$

where $S(\beta)$ is the sum-of-squares function.

When this function is minimized, the vector b, the least squares estimator of β, is determined. Two important properties for an

estimator are unbiasedness and efficiency. An estimator is unbiased if its expected value is equal to the value of the parameter. This means that for all possible samples of size n, the mean value of the estimates obtained would be equal to the value of the parameter. Even though an estimator is unbiased, an estimate from one set of observations will in general not be exactly equal to the population parameter because sampling error is introduced when all observations of the population are not included in the estimation process. An estimator is efficient if it has the minimum variance among all estimators in its class. Again if all possible estimates from all possible samples could be obtained, the variance of the estimator is a measure of the dispersion of the distribution of these estimates. A minimum variance is desired because an estimate obtained from the sample actually observed would generally be close to an estimate obtained from any other sample which could have been selected. From this vector b, it can be calculated,

$$Y_i = f(X_i, b) \tag{16}$$

which becomes an estimate of the expected value of the dependent variable. The resulting error is given by the residuals which can be denoted as,

$$e_i = Y_i - \hat{Y}_i \tag{17}$$

Linear least squares

The linear model may be written as,

$$Y_i = X \cdot \beta + \epsilon_i, \quad i = 1, \ldots, n \tag{18}$$

where

Y_i = the dependent variable, is continuous,

X = the matrix of independent variables, which are considered fixed and measures without error, is nxp and of full rank,

β = the vector of p unknown parameters, and,

ϵ_i = the stochastic elements known as the error terms, are usually assumed to be independent and identically distributed (i.e., i.i.d.) random variables as well as often assumed to be normally distributed with mean zero and variance σ^2, or $N(0, \sigma^2)$.

Five assumptions are necessary for the least squares estimators to be B.U.L.E. (Best Linear Unbiased Estimator) and to rely upon the t-test or normal distributions for confidence intervals. These assumptions are as follows:

1. Normality: ϵ_i is normally distributed.

2. Zero mean: $E(\epsilon_i) = 0$.

3. Homoscedasticity: $Var(\epsilon_i) = \sigma^2$.

4. Nonautocorrelation: $E(\epsilon_i, \epsilon_j) = 0$, for $i \neq j$.

5. Nonstochastic X: X_i is a nonstochastic variable with values fixed in repeated samples.

The following theoretical effects may exist if violations occur:

Non-normality of error term   If the assumption that the
disturbance is normally distributed is violated, the least squares
estimators of the linear model are still B.L.U.E., since this property
is independent of the form of the parent population. This means that
even without the assumption of normality, the least squares estimators
are unbiased and have the smallest variance among all linear unbiased
estimators of the respective parameters; however, they are no longer
efficient. The confidence intervals depend crucially on the assumption
of normality. Without the assumption of normality, the least squares
estimators are not normally distributed in small samples, and the
confidence limits no longer apply.

Non-zero expected value of error term   If the relationship
between the independent variables and dependent variable has not been
correctly specified, the mean disturbance may be non-zero.

Heteroscedasticity of error term   When the assumption of
constant variance is violated, the distributions and expectations of
the least squared estimates for the coefficients do not change. They
are still linear and unbiased estimators with the property of
consistency. The problem arises in the estimation of the variance of
these coefficient estimates. If we assume a constant variance of e_i
when, in fact, the variance changes for different observations, then we
are not making use of all the possible information. This is an
indication that the estimators are not efficient. That is, the
variances of the coefficients determined through the ordinary least

squares method are greater than variances determined by some other estimating procedure which uses the additional information about the changing variance of the disturbance term. Heteroscedasticity often causes the ordinary least squares estimator of the variance for the coefficients to be biased downward, i.e., the estimated value of the standard error of an estimated coefficient is lower than it would be otherwise [59]. The effect of this downward bias is that the t and F statistics used for testing the significances of the coefficients will be overstated.

<u>Autocorrelation</u> <u>of</u> <u>error</u> <u>term</u>   Violation of the assumption of independent error terms arises frequently when time series data are used, especially if the time interval between observations is small. With this violation of the autocorrelation, the ordinary least squares estimators are affected by misrepresentation of the variance-covariance matrix of disturbances. Baillie [60] showed through mathematical computation that by ignoring autocorrelation the usual ordinary least squares estimate of the variance of the error term would be biased and would generally underestimate the true prediction of the mean squared error. He supported this result by simulations in which autocorrelation is ignored when estimating the models by ordinary least squares method. The mean square error of the correct model is then compared with the mean square error of the simulated models and found to be consistently lager. The bias in the estimate of the standard error of the coefficient is downward if the autocorrelation is positive and upward if the autocorrelation is negative.

<u>Contemporaneous correlation</u>   When the assumption of independence between the independent variable and the error term is violated, the estimates of the coefficients of the linear model are biased. The problem of contemporaneous correlation is most often linked with either error in measurement of the independent variable or with simultaneity of the relation between the dependent and independent variable. One of the major problems of errors in measurement or simultaneity is that the disturbance term becomes a composite of two or more different errors and the total error variance cannot be decomposed unambiguously. It may be a random error associated with the specification plus an error of measurement. Even in large samples where the relevant sampling moment approaches the true variance, this true covariance is still nonzero and the bias remains. The estimates of the parameters do not have the property of consistency.

The sum of squares of deviations from the true linear relationship, using the above assumptions, becomes,

$$S(\beta) = \sum_{i=1}^{n} \epsilon_i^2$$

$$= \sum_{i=1}^{n} [Y_i - y_i]^2 \tag{19}$$

In order to minimize this function, (19) can be differentiated with respect to the vector β and set equal to zero; an exact answer to these questions can be found since normal equations are linear. The resultant matrix of normal equations is,

$$b = (X'X)^{-1} \cdot (X'Y) \tag{20}$$

where $E(b) = \beta$ and $Var(b) = \sigma^2 \cdot (X'X)^{-1}$ [61].

Nonlinear least squares

If the model being fitted is not linear in its parameters and can not be rendered linear by a transformation, then the estimation of the parameters becomes a more difficult task. The general nonlinear model is given by the equation,

$$Y_i = f(X_i, \beta) + \epsilon_i, \quad i = 1,\ldots,n \tag{21}$$

where

Y_i = the dependent variable for the ith of n observations,

X_i = a row vector of scores for p independent variable,

β = the vector of p unknown population parameters, and

ϵ_i = the error for the ith observation, which is i.i.d.,

$N(0, \sigma^2)$.

Under the assumption of normally and identically distributed independent errors, the general nonlinear model in equation (21) has likelihood,

$$L(\beta, \sigma^2) = (2\pi\sigma^2)^{-(n/2)} \exp[- \frac{1}{2\sigma^2} S(\beta)] \tag{22}$$

where $S(\beta)$ is the sum-of-squares function, such as,

$$S(\beta) = \sum_{i=1}^{n} [Y_i - f(X_i, \beta)]^2 \tag{23}$$

As in the case of the linear model, the likelihood is maximized when the function of $S(\beta)$ is minimized. To derive estimating equations for the nonlinear model, differentiate $S(\beta)$ with respect to the vector β,

$$\frac{\partial S(\beta)}{\partial \beta} = -2(\frac{\partial f(X_i, \beta)}{\partial \beta}) \sum_{i=1}^{n} [Y_i - f(X_i, \beta)] \qquad (24)$$

Setting these partial derivatives to zero and replacing the unknown parameters β with the estimator b, produce the nonlinear least squares normal equations. Because the estimating equations (24) arising from a nonlinear model are in general themselves nonlinear, their solution is often difficult. If it is not possible to solve algebraically, iterative methods are then employed in most cases [62]. There are several methods for obtaining nonlinear least squares estimate, such as, steepest descent method [63], Gauss-Newton method [64], and the Marquardt's compromise method [65].

One of the assumptions for the nonlinear model is that the error terms, $\epsilon_i \sim$ i.i.d. $N(0, \sigma^2)$, as well as, being additive in the model; this implies that the error terms have equal variances and are uncorrelated. According to Draper and Smith [62], growth data do not always satisfy the least squares assumptions. With such time series data, there is little reason to believe a priori that the error terms are uncorrelated. The closer in proximity of the time the two data points occur, the greater the likelihood for positive correlation.

If the assumption of homoscedasticity is rejected, the method of weighted least squares can be applied in an attempt to alleviate the problem. The observations are divided into subgroups and s^2 of each subgroup is used as an estimate for σ^2. Then the original observations within each group are weighted inversely according to the standard error of estimate for that group.

If the assumption of autocorrelation is rejected, the Cochran and Orcutt's least squares method [66] can be used to transform the data so as to reduce the autocorrelation present in the data.

Discounted Least Squares Method

The difficulty with the ordinary least squares method, whether linear or nonlinear, is the assumption that all data are equally weighted. When modelling time series data, this assumption will probably be deemed inappropriate. In addition, the assumption of the β being constant throughout time may also be inappropriate. As time progresses, it may be the case that the relationship between Y and X may be shifting, such that β is no longer constant. Gilchrist [67] referred to this situation as a local model in contrast to a constant global model. When dealing with such data, the equal weighting of the data may be no longer sensible.

If the assumption is made that the model being employed is only locally constant so that parameter changes are permissible, more weight could be given to recent observations, and the past observations could

be discounted. This reasoning motivated Brown [68] to consider a discount factor in the least squares criterion. In discounted least squares or general exponential smoothing, the parameter estimates are determined by minimizing,

$$D(\beta) = \sum_{i=0}^{n-1} w^{n-i} [Y_i - f(X_i, \beta)]^2. \tag{25}$$

The constant w ($0 < w \leq 1$) is a discount factor that discounts past observations exponentially. Brown suggested that w be chosen such that $0.70 < w^m < 0.95$, where m is the number of estimated parameters in the model. If w is small, more weight is given to the more recent observations and the information from previous periods is heavily discounted. If w is set to one, the solution to the minimization problem becomes the normal equations of the ordinary least squares problem.

Once w has been specified, a nonlinear least squares methods can be utilized to minimize the function. The equation (25) can be rewritten as,

$$D(\beta) = \sum_{i=0}^{n-1} [w^{(n-i)/2} Y_i - w^{(n-i)/2} f(X_i, \beta)]^2 \tag{26}$$

where $w^{(n-i)/2}$ is inserted inside the residual term and treated as a constant. That is, the same nonlinear least squares method can be employed using $Y^* = w^{(n-i)/2} \cdot Y$ and $f^*(X_i, \beta) = w^{(n-i)/2} \cdot f(X_i, \beta)$.

CHAPTER VI. SELECTION OF GROWTH MODELS

The various forms of the growth curves that are presently employed in the technological forecasting methods were reviewed in Chapter Four by the degree of skewness. It was demonstrated that some of the models are mathematical transformations of one another.

Since many growth models have formulations with exponential functions, logarithmic transformations are used to linearized these curves. For example, the deterministic form of the Pearl growth curve can be transformed to obtain the Fisher-Pry and the Bass error free models and can be converted into a linearized function. For the Gompertz curve, the log of the negative log, $-\ln[-\ln(Y/L)]$, would result in a linearized function. For the Weibull curve, the log of the negative log, $\ln\{-\ln[(L-Y)/L]\}$, would also result in a linearized function. But the addition of the stochastic element, $\epsilon(t)$, to the deterministic form of these models results in different error structure and, therefore, different growth models. The calculation of variance for a simple model is not a difficult task. For a linearized growth model, however, an approximate technique is employed to calculate the variance.

Error Structure of Growth Models

If the general linear model is given,

$$Y = \beta_0 + \beta_1 X \qquad\qquad (27)$$

where $X \sim (\mu, \sigma^2)$,

then, knowing $Var(X) = \sigma^2$, one can solve for the variance of Y,

$$Var(Y) = \beta_1{}^2 \cdot [Var(X)]$$

$$= \beta_1{}^2 \cdot \sigma^2 \tag{28}$$

This approach can not be used when Y is a nonlinear function of X. However, if $Y = g(X)$ is specified, using the Taylor series expansion [69], Y can be rewritten as,

$$g(X) = g(\mu) + (X-\mu)g'(\mu) + \frac{1}{2}(X-\mu)^2 g''(\mu)$$

$$+ \ldots + \frac{1}{n!}(X-\mu)^n g^n(\mu) + \ldots \tag{29}$$

where $g^n(\mu)$ is the nth derivative and is evaluated at μ. Taking the expectation of $g(X)$,

$$E[g(X)] = g(\mu) + \frac{1}{2}g''(\mu)E(X-\mu)^2 + \text{remainder} \tag{30}$$

where $E(X-\mu) = 0$, and $E(X-\mu)^2 = Var(X) = \sigma^2$, then, equation (30) can be rewritten as,

$$E[g(X)] = g(\mu) + \frac{1}{2}g''(\mu)\sigma^2 + \text{remainder} \tag{31}$$

The variance can be specified as,

$$Var[g(X)] = E\{g(X) - E[g(X)]\}^2$$

$$= E[g^2(X)] - \{E[g(X)]\}^2 \tag{32}$$

from equation (29),

$$g^2(X) = g^2(\mu) + \{[g'(\mu)]^2 + g(\mu)g''(\mu)\}(X-\mu)^2 + \text{remainder} \qquad (33)$$

and

$$E[g^2(X)] = g^2(\mu) + \{[g'(\mu)]^2 + g(\mu)g''(\mu)\}\sigma^2 + \text{remainder},$$

$$= g^2(\mu) + \frac{1}{2}[g^2(\mu)]''\sigma^2 + \text{remainder} \qquad (34)$$

Hence,

$$\text{Var}[g(X)] \simeq g^2(\mu) + \frac{1}{2}[g^2(\mu)]''\sigma^2$$

$$- [g(\mu) + \frac{1}{2}g''(\mu)\sigma^2]^2 + \text{remainder},$$

$$= \sigma^2\{\frac{1}{2}[g^2(\mu)]'' - g(\mu)g''(\mu)\} + \text{remainder} \qquad (35)$$

from equation (33),

$$[g'(\mu)]^2 = \frac{1}{2}[g^2(\mu)]'' - g(\mu)g''(\mu) \qquad (36)$$

So,

$$\text{Var}(Y) = \text{Var}[g(X)]$$

$$\simeq \sigma^2 \cdot [g'(\mu)]^2 \qquad (37)$$

This implies that if the mean and variance of X are known, then the variance of Y, which is a nonlinear function of X, can be approximated by the variance of X times the sequence of the derivative of g(X) when evaluated at μ. This approximation technique for calculating the variance of Y can be applied to the six growth curves that were selected by the degree of skewness and the transformation of the functions: the Pearl growth curve, the linearized Fisher-Pry model, the Gompertz growth curve, the linearized Gompertz growth curve, the Weibull growth curve, and the linearized Weibull growth curve.

Pearl growth curve

If the Pearl growth curve is specified as,

$$Y(t) = \frac{L}{1 + ae^{-\beta t}} + \epsilon(t) \tag{38}$$

where

$\epsilon(t) \sim$ i.i.d. $N(0, \sigma^2)$,

then $E[Y(t)] = 1/[1+\exp(-\beta t)]$ and $Var[Y(t)] = Var[\epsilon(t)] = \sigma^2$. This result indicates that the variance of the observed $Y(t)$ is constant over time.

Linearized Fisher-Pry model

If the linear version of the Fisher-Pry model is specified as,

$$U(t) \equiv \ln \left(\frac{Y(t)}{L - Y(t)}\right) = \beta_0 + \beta_1 t + \epsilon(t) \tag{39}$$

where

$$\epsilon(t) \sim \text{i.i.d. } N(0, \sigma^2),$$

then $\text{Var}[U(t)] = \text{Var}[\epsilon(t)] = \sigma^2$. But it is the variance of $Y(t)$ that needs to be identified. Solving for $Y(t)$,

$$Y(t) = g[U(t)] = \frac{L}{1 + e^{-U(t)}} \tag{40}$$

Taking the first derivative,

$$g'[U(t)] = \frac{Le^{-U(t)}}{[1 + e^{-U(t)}]^2} \tag{41}$$

Let $u_t = E\{g[U(t)]\} = L/\{1+\exp[-U(t)]\}$, then the variance of $Y(t)$ becomes, by equation (37),

$$\text{Var}[Y(t)] \simeq \sigma^2 \{g'[U(t)]\}^2$$

$$= \sigma^2 L^2 \left| \frac{Le^{-2U(t)}}{[1 + e^{-U(t)}]^4} \right| \quad \text{at } g[U(t)]=u_t$$

$$= \frac{\sigma^2 u_t^2 (L - u_t)^2}{L^2} \tag{42}$$

Let $c = (\text{variance})^{0.5}/\sigma$, then equation (42) can be specified as,

$$c = u_t - \frac{1}{L} u_t^2, \text{ and}$$

$$\frac{dc}{du_t} = 1 - \frac{2}{L} u_t \tag{43}$$

When the equation (43) is set equal to zero, $u_t = L/2$, which indicates that the maximum of variance function occurs at inflection point, because,

$$\frac{d^2}{du_t{}^2} = -\frac{2}{L} < 0 \tag{44}$$

This implies that the variance of the $Y(t)$ is not a constant function and has the greatest variability at the inflection point of the growth curve.

Gompertz growth curve

If the original form of the Gompertz growth curve is specified as,

$$Y(t) = L \cdot e^{-G \cdot e^{-kt}} + \epsilon(t) \tag{45}$$

where

$\epsilon(t) \sim$ i.i.d. $N(0, \sigma^2)$,

then $E[Y(t)] = L \cdot \exp[-G \cdot \exp(-kt)]$ and $Var[Y(t)] = Var[\epsilon(t)] = \sigma^2$. This result indicates that the variance of the observed $Y(t)$ is constant over time.

Linearized Gompertz growth curve

The linear version of the Gompertz growth curve can be analyzed in the same manner as the linearized Fisher-Pry model. If the model is specified as,

$$V(t) = -\ln[-\ln(\frac{Y(t)}{L})] = \beta_0 + \beta_1 t + \epsilon(t) \tag{46}$$

where

$$\epsilon(t) \sim i.i.d. \ N(0, \sigma^2),$$

then the model can be rewritten as,

$$Y(t) \equiv g[V(t)] = L \cdot e^{-e^{-V(t)}} \tag{47}$$

Taking the first derivative,

$$g'[V(t)] = L \cdot \exp\{-[V(t) + \exp(-V(t))]\} \tag{48}$$

Let $v_t \equiv E\{g[V(t)]\} = L \cdot \exp\{-\exp[-V(t)]\}$, then the variance of $Y(t)$ becomes,

$$Var[Y(t)] \simeq \sigma^2 \{g'[V(t)]\}^2$$

$$= \sigma^2 L^2 \ |\exp\{-2[V(t)+\exp(-V(t))]\}|$$
$$\text{at } g[V(t)]=v_t$$

$$= \sigma^2 v_t^2 \ [\ln\frac{L}{v_t}]^2 \tag{49}$$

Let $c = (variance)^{0.5}/\sigma$, then equation (49) can be specified as,

$$c = v_t \ln L - v_t \ln v_t, \text{ and}$$

$$\frac{dc}{dv_t} = \ln L - \ln v_t - 1 \tag{50}$$

When the equation (50) is set equal to zero, $v_t = L/e$, which indicates that the maximum of variance function occurs at the inflection point.

This implies that the variance of the Y(t) is not a constant function and has the greatest variability at the inflection point.

Weibull growth curve

The Weibull growth curve can be analyzed in the same manner as the Gompertz growth curve. If the original form of the Weibull growth curve is specified as,

$$Y(t) = L - L \cdot e^{-(\frac{t}{a})^{\beta}} + \epsilon(t) \qquad (51)$$

where

$\epsilon(t) \sim$ i.i.d. $N(0, \sigma^2)$,

then, $Var[Y(t)] = Var[\epsilon(t)] = \sigma^2$. Again, the variance of the observed Y(t) is constant over time.

Linearized Weibull growth curve

For the linear version of the Weibull growth curve, the variance of Y(t) is no longer a constant function. Utilizing the Taylor series expansion, the variance can be specified in the following fashion. If the model is specified as,

$$W(t) \equiv \ln[-\ln(\frac{L - Y(t)}{L})] = \beta_0 + \beta_1 t + \epsilon(t) \qquad (52)$$

where

$\epsilon(t) \sim$ i.i.d. $N(0, \sigma^2)$,

then the model can be rewritten as,

$$Y(t) = g[W(t)] = L - L \cdot e^{-e^{W(t)}} \tag{53}$$

Taking the first derivative,

$$g'[W(t)] = L \cdot \exp\{-[\exp(W(t)) - W(t)]\} \tag{54}$$

Let $w_t = E\{g[W(t)]\} = L - L \cdot \exp\{-\exp[W(t)]\}$, then the variance of $Y(t)$ becomes,

$$Var[Y(t)] \simeq \sigma^2 \{g'[W(t)]\}^2$$

$$= \sigma^2 L^2 \left| \exp\{-2[\exp(W(t)) - W(t)]\} \right| \quad \text{at } g[W(t)] = w_t$$

$$= \sigma^2 (L - w_t)^2 \left[\ln\left(\frac{L}{L - w_t}\right) \right]^2 \tag{55}$$

For the Pearl growth curve, the Gompertz growth curve, and the Weibull growth curve, the errors have zero mean and a constant variance over time. However, transformed models like the linearized Fisher-Pry model, the linearized Gompertz growth curve, and the linearized Weibull growth curve have increasing variance from time zero to that point at which inflection occurs. It can be assumed that if the variance of error over time is increasing, then a transformation of observed data is appropriate.

Test for Assumptions of Error Structure

In order to select the appropriate growth curve for a set of time series data, it is suggested to test assumptions of error terms.

Normality of error terms

Hypothesis

H_1: The error terms of the growth model are normally distributed.

H_A: The error terms of the growth model are not normally distributed.

Test   If the observed data are less than 51, the Shapiro-Wilk test is used to test this hypothesis [70]. This test is the ratio of the best estimator of the variance to the usual corrected sum-of-squares estimator of the variance. This statistic must be greater than 0 and less than or equal to 1. If the observed data are greater than 50, the Kolmogorov goodness-of-fit test is used to test this hypothesis [71]. This test measures the largest vertical distance between the cumulative empirical and cumulative hypothesized distribution function, and compares this statistic to calculated table value. This test for normality is chosen rather than the chi-square goodness-of-fit test because the Kolmogorov test is exact even for small samples, while the chi-square test assumes that the number of observations is large enough so that the chi-square distribution provides a good approximation of the distribution of the test statistic.

The procedure for testing the residuals of each model for
normality is provided by UNIVARIATE NORMAL procedure included in SAS
(Statistical Analysis System) package of computer programs. After each
growth model is determined, the error terms are calculated and added to
the SAS data set which already contains all the data. The Kolmogorov
test is performed on these calculated residuals. The procedure shows
the probability that there would be a smaller value than the test value
if the residuals followed the hypothesized distribution.

Decision rule   Reject H_1, if the probability of a smaller
statistic calculated by the UNIVARIATE procedure is less than 0.05.

Homoscedasticity of error terms

Hypothesis

> H_2: The error terms of the growth model have
> constant variance.

> H_A: The error terms of the growth model do not have
> constant variance.

Test   The Goldfeld-Quandt test is used to test this hypothesis
[72]. This test divides the observations into two subgroups (of equal
size if n is even and a difference in size by one if n is odd),
calculates the sum of squares of residuals in each model, and uses the
following F-distributed test statistic to test the null hypothesis of
homoscedasticity against a two sided alternative that the variance of
the distribution term is different for the two subgroups:

$$F(n_1-k, n_2-k) = \frac{\displaystyle\sum_{i=1}^{n_1} \frac{e_i{}^2}{n_1-k}}{\displaystyle\sum_{i=1}^{n_2} \frac{e_i{}^2}{n_2-k}} \tag{56}$$

where

n_1 = number of observations in subgroup 1,

n_2 = number of observations in subgroup 2,

k = number of parameters estimated.

An F-distributed ratio of two chi-square distributed random variables could be used to test if difference between the variance of two separate groups of residuals is significant. Under the usual assumptions, the test on the difference between variance uses a statistic,

$$F(n_1-k, n_2-k) = \frac{s_1{}^2}{s_2{}^2} \tag{57}$$

where $s_i{}^2$ is the unbiased sample estimate of the variance. However, this test is not used because the estimates being used are based on the sum of squares of subgroups of residuals from the same growth model. These are not statistically independent since residual depends upon the estimate b which depends upon all residuals. Thus, this statistic is not a ratio of two independent chi-squares distributed variables as required for the F-distribution. In performing the Goldfeld-Quandt test, each set of observed data is split into two groups; the earlier

years in group 1 and the later years in group 2. The ordinary least squares method is performed on each group independently. The ratio of two mean square error is calculated and compared to a table F-value. To detect heteroscedasticity, however, it is useful to plot residuals against fitted dependent variable values [61].

Decision rule   Reject H_2, if the calculated F-statistic is greater than the table F-value at the $a = 0.05$ level of significance.

Nonautocorrelation of error terms

Hypothesis

H_3: The error terms of the growth model are
independent of one another.

H_A: The error terms of the growth model are
correlated with one another.

Test   The Durbin-Watson test statistic (D) is used to test this hypothesis [73]. The test statistic is,

$$D = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2} \tag{58}$$

Its distribution is tabled for selected values of the number of observations (n) and the number of independent variables which excludes the intercept. A value of D close to 0 or 4 would indicate positive or negative autocorrelation respectively while a value close to 2 would

indicate independence. PROC GLM of SAS programs includes an option which calculates the Durbin-Watson statistic for linear estimation models. PROC MATRIX of SAS programs was used to calculate the Durbin-Watson statistic for nonlinear estimation models. The list of programs is given in Appendix D.

Decision rule   Reject H_3, if $D < d_L$ or if $D > 4-d_L$ at level 2α. Do not reject H_3, if $d_U < D < 4-d_U$ at level 2α. Otherwise, the test is inconclusive, where d_L and d_U are values in the Durbin-Watson table [74, see Appendix B., Table B.9].

CHAPTER VII. STATISTICAL ANALYSIS AND EMPIRICAL RESULTS

This chapter represents the results after applying the appropriate statistical procedures and tests to the actual data. Six mathematical forms of an S shaped growth curves are considered as models for yearly percentages of technological attainment. The upper limit is set at 100 percent attainment. Given that $Y(t)$ is the cumulative percentage of technological attainment at time t (i.e., penetration level achieved at time t), the three nonlinearized growth models employed are,

1. Pearl growth curve

$$Y(t) = \frac{1.0}{1 + ae^{-\beta t}} + \epsilon(t) \tag{59}$$

2. Gompertz growth curve

$$Y(t) = e^{-G \cdot e^{-kt}} + \epsilon(t) \tag{60}$$

3. Weibull growth curve

$$Y(t) = 1.0 - e^{-(\frac{t}{a})^{\beta}} + \epsilon(t) \tag{61}$$

where

$\epsilon(t) \sim$ i.i.d. $N(0, \sigma^2)$.

The three linear version of growth models are,

1. Linearized Fisher-Pry model

$$\ln\left(\frac{Y(t)}{1 - Y(t)}\right) = \beta_0 + \beta_1 t + \epsilon(t) \tag{62}$$

2. Linearized Gompertz growth curve

$$-\ln\{-\ln[Y(t)]\} = \beta_0 + \beta_1 t + \epsilon(t) \tag{63}$$

3. Linearized Weibull growth curve

$$\ln\{-\ln[1.0 - Y(t)]\} = \beta_0 + \beta_1 t + \epsilon(t) \tag{64}$$

where

$\epsilon(t) \sim \text{i.i.d. } N(0, \sigma^2)$.

Data

The data used in this study are twenty-two historical growth cases from various industries. The main requisite for inclusion in the set was that any case has at least two points before the five percent penetration level, an increasing number of points through ten percent, twenty-five percent, fifty percent, and several points beyond the seventy-five percent penetration level. This was because the analysis was designed to check for forecasting ability at each and every one of those levels.

In all cases except two, it was assumed that penetration would go to a hundred percent. In the case of "percentage of households in the U.S. with a radio receiver" and "percentage of households in the U.S. with a television set", the adoption seemed to have leveled off at about ninety-nine percent for a radio receiver and ninety-eight percent for a television set. Those levels were then assumed to be upper

limits and the data values were adjusted so that each values was taken as a percentage of its respective upper limit. A list of the cases and their sources is given in Appendix E.

Comparison of the Models

Prediction achievement is the criteria which will be used to determine if any particular model was or a group of models were dominantly superior to other models as forecasters of technological growth at different penetration levels. A measure of mean square error (m.s.e.) is used to determine the adequacy of the fit. The smallest mean square error may be used as an indication of which model to use. Since the fitted data of linearized models must be transformed back into y_t values, the R^2 (coefficient of multiple determination) provided in the computer fitting process cannot be utilized; for this reason a modified mean square error (i.e., mean estimate error) is used to align for model comparisons,

$$\text{m.e.e.} = \frac{\sum\limits_{t=1}^{N} (Y_t - y_t)^2}{N} \cdot 1000 \tag{65}$$

where Y_t is the actual penetration achieved at time t, y_t is the predicted penetration at time t, N is the number of terms in the series, and multiplying by 1000 is only to avoid working with very small numbers. This procedure is not unique and it has been used by

other investigators such as Nagar [75] and Eilon et al. [76]. A forecasting error was defined analogously. For example, using one of growth models at the ten percent penetration level to forecast case 22, all points for case 22 up to and including the ten percent point, if it was one of the points, would be used at the estimation stage. All points beyond the ten percent level would be forecast and the sum of the squared forecast error obtained. That sum would then be divided by the number of points forecast to obtain the average squared forecast error (i.e., mean forecast error). Tables of such errors, on a case by case basis, at each level of fitting and forecasting errors are given in Appendix F and Appendix G.

Comparison of the models is accomplished by testing the following hypothesis:

H_4: No significant difference exists among the fitting errors of technological growth models at each penetration level.

H_A: At least one significant difference exists between the fitting errors of technological growth models at each penetration level.

H_5: No significant difference exists among the forecasting errors of technological growth models at each penetration level.

H_A: At least one significant difference exists between the forecasting errors of technological growth models at each penetration level.

To test these hypotheses and determine if any one growth model outperforms any other of the growth models at each penetration level, an analysis of variance (ANOVA) is performed. An F-test provides evidence that a difference may exist among means (mean estimate error or mean forecast error) of several groups. However, if there are k different means being compared, there are $k(k-1)/2$ potential differences, and the F-test dose not indicate between which means these differences may be. Therefore, a modified Tukey's test [77] is performed to test all comparisons among means. The test is made by computing a difference, D, which is significant at the 5% level, when comparing it with the $k(k-1)/2$ sample differences in the experiment. D is the product of a square root of s^2/n and a factor, Q, where s^2 is the estimate of the variance within the groups, and Q is given in the table of "studentized range" [78].

If any sample differences exceed the calculated D-value, then a significant difference may exist between the means of those two groups and may indicate that one growth model may perform significantly different from another growth model.

In order to perform a parametric analysis of variance such as Tukey's test, several assumptions about the data are necessary. These assumptions include:

1. Homogeneity of variances between populations,

2. Normality of error terms,

3. Additivity,

4. Independence of error terms.

However, if these assumptions are violated by the data, a nonparametric analysis of variance would be more appropriate. Therefore, the same data are tested using the nonparametric Kruskal-Wallis test [79] available in the SAS package of computer programs by NPARIWAY procedure. In this test, k random samples are obtained from k possibly different populations and the null hypothesis that all of the populations are identical is tested against the alternative that some of the populations tend to furnish greater observed values than other populations.

The Kruskal-Wallis test is chosen rather than the median test since the former uses more information contained in the observations than does the latter. The Kruskal-Wallis test statistic is a function of the ranks of the observations in the combined sample while the median test statistic is dependent only on the knowledge of whether the observations were below or above the grand median. For this reason the Kruskal-Wallis test is usually more powerful than the median test. All statistical significance tests were performed at the 95% confidence level.

Results for Assumptions of Error Structure

Normality of error terms

Table 2 contains the summarized results of tests for normality of error terms using the six growth models calculated from the historical growth cases.

TABLE 2. Summary of tests for normality of error terms

	LFP	LGZ	LWB	PL	GZ	WB	Total
Not rejected	12	15	7	11	14	15	74
Rejected	10	7	15	11	8	7	58

Not rejected indicates that there is not sufficient evidence to show
that the error terms are not normally distributed.
Rejected indicates that there is sufficient evidence to show that
the error terms are not normally distributed.

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

This table is derived from the individual case results contained
in Appendix H, which includes the a-values (level of significance) for
normality test. As shown in Table 2, nonnormality of the distribution
of error terms exists in many of the growth models which are used for
predictive purposes. This result illustrates that the applied
transformations (i.e., linearized models) did not remove the
nonnormality of error terms in many cases.

The violation of the assumption of error term normality will
affect confidence intervals established for predictions from the final
model. Nonnormality of error terms will cause intervals of given
confidence levels to be calculated as smaller than they would have been
if the true distribution were used in the determination of the
interval.

Homoscedasticity of error terms

Table 3 contains the summarized results of the tests for constant variance of the error terms using the various growth models calculated from the historical growth cases.

TABLE 3. Summary of tests for homoscedasticity

	LFP	LGZ	LWB	PL	GZ	WB	Total
Not rejected	11	7	2	9	9	7	45
Rejected	11	15	20	13	13	15	87

Not rejected indicates that there is not sufficient evidence to show that the error terms do not have constant variance.
Rejected indicates that there is sufficient evidence to show that the error terms do not have constant variance.

 Key: LFP = linearized Fisher-Pry model
      LGZ = linearized Gompertz growth curve
      LWB = linearized Weibull growth curve
      PL = Pearl growth curve
      GZ = Gompertz growth curve
      WB = Weibull growth curve

This table is derived from the individual case results contained in Appendix I, which includes the F-values for homoscedasticity test. The results from Table 3 suggest that the applied linearized models did not remove the violation of heteroscedasticity. The presence of heteroscedasticity causes the variance of the estimates to be biased upward. At a given level of reliability, this may cause a wider

confidence interval to be constructed than would have been constructed had there been no bias.

Autocorrelation of error terms

Table 4 contains the summarized results of the tests for the independence of the error terms using the various growth models calculated from the historical growth cases.

TABLE 4. Summary of tests for autocorrelation

	LFP	LGZ	LWB	PL	GZ	WB	Total
Not rejected	0	1	1	1	3	2	8
Rejected	22	21	21	21	19	20	124

Not rejected indicates that there is not sufficient evidence to show that the error terms are not independent.
Rejected indicates that there is sufficient evidence to show that the error terms are not independent.

Key: LFP = linearized Fisher-Pry model
    LGZ = linearized Gompertz growth curve
    LWB = linearized Weibull growth curve
    PL = Pearl growth curve
    GZ = Gompertz growth curve
    WB = Weibull growth curve

This table is derived from the individual case results contained in Appendix J, which includes the Durbin-Watson test statistic values. Since time series data are being used, a priori autocorrelation is suspected to be a problem. Table 4 indicates that growth models experience a significant degree of autocorrelation.

As shown in tables, the violation of the assumption of no autocorrelation appeared to have the most serious consequences, followed by heteroscedasticity, and then nonnormality. Interrelationships may exist among assumption violations of error terms. For example, the test for homoscedasticity of error terms is very sensitive to nonnormality. Sometimes the removal of one violation may be accomplished only at the expense of creating another violation. Therefore, a transformation is not performed on a model when the result is to reduce the effect of one violation and increase the effect of a more serious violation.

Results for Comparison of the Fitting Ability

Results of Tukey test

The comparison is performed using Tukey's test which is an analysis of variance among the fitting errors of growth models. Table 5 contains the analysis of variance results when Tukey's test was applied to the relative fitting errors of growth models for twenty-two cases at 5% penetration level. The factor Q is obtained from the "studentized range" table from Biometrika [78] for six treatments and 126 degree of freedom. D represents the largest difference between the means of any two treatments that may exist and still be considered sampling error rather than a difference between treatment means.

Table 6 contains the difference between sample means of the six growth models performed at 5% penetration level.

TABLE 5. ANOVA of fitting error at 5% level

source of variation	d.f.	sum of squares	mean square
model	5	0.00141801	
error	126	0.01615207	0.00012819
total	131	0.01757008	

$Q = 4.093$
$s = (0.00012819/22)^{1/2} = 0.002414$
$D = Q \cdot s = 0.00988$

TABLE 6. Fitting differences between models at 5% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(0.015)	−	0.006249	0.007630	0.008797	0.008964	0.009734
LGZ(0.009)		−	0.001381	0.002548	0.002715	0.003485
LWB(0.007)			−	0.001167	0.001333	0.002103
PL(0.006)				−	0.000166	0.000936
GZ(0.006)					−	0.000770
WB(0.005)						−

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

The bracketed number after the name of model is the mean for the twenty-two cases of the mean estimate error using each growth model as described in equation (65). As can be seen from Table 6, none of the difference exceeds the calculated Tukey's D of 0.00988; therefore, H_4

(there exists no significant difference among the fitting errors of growth models) hypothesis cannot be rejected for the 5% penetration level at the 5% significance level. There is not sufficient evidence to show that a true difference exists between any of the growth models at 5% penetration level, and those differences that are shown in Table 6 are probably due to the sampling error.

Table 7 contains the analysis of variance results when Tukey's test was applied to the relative fitting errors of growth models at 10% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

TABLE 7. ANOVA of fitting error at 10% level

source of variation	d.f.	sum of squares	mean square
model	5	0.03888254	
error	126	0.39649599	0.00314679
total	131	0.43537853	

$$Q = 4.093$$
$$s = (0.00314679/22)^{1/2} = 0.011960$$
$$D = Q \cdot s = 0.04895$$

Table 8 contains the difference between sample means of the six growth models which are compared to Tukey's D of 0.04895.

The H_1 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the Weibull growth curve. Since this difference

TABLE 8. Fitting differences between models at 10% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(0.076)	–	0.03209	0.03359	0.04537	0.04764	0.05127*
LGZ(0.044)		–	0.00150	0.01328	0.01555	0.01918
LWB(0.042)			–	0.01178	0.01405	0.01768
PL(0.030)				–	0.00227	0.00590
GZ(0.028)					–	0.00362
WB(0.025)						–

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve
      * = indicates significant difference exists between models

is too large to be due to sampling error alone, the linearized Fisher-Pry model appears to have a larger fitting error than the Weibull growth curve at 10% penetration level.

Table 9 contains the analysis of variance results when Tukey's test was applied to the relative fitting errors of growth models at 25% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

Table 10 contains the difference between sample means which are compared to Tukey's D of 0.34389.

The H_4 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other growth models. This result indicates

TABLE 9. ANOVA of fitting error at 25% level

source of variation	d.f.	sum of squares	mean square
model	5	3.43363439	
error	126	19.56926028	0.15531159
total	131	23.00289467	

$$Q = 4.093$$
$$s = (0.15531159/22)^{1/2} = 0.084019$$
$$D = Q \cdot s = 0.34389$$

TABLE 10. Fitting differences between models at 25% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(0.62)	–	0.3640*	0.3746*	0.4317*	0.4612*	0.4672*
LGZ(0.26)		–	0.0106	0.0677	0.0972	0.1032
LWB(0.24)			–	0.0571	0.0866	0.0926
PL(0.19)				–	0.0295	0.0355
GZ(0.16)					–	0.0060
WB(0.15)						–

Key: LFP = linearized Fisher-Pry model
    LGZ = linearized Gompertz growth curve
    LWB = linearized Weibull growth curve
    PL = Pearl growth curve
    GZ = Gompertz growth curve
    WB = Weibull growth curve
     * = indicates significant difference exists between models

that the linearized Fisher-Pry model appears to have a larger fitting

error than the other growth models at 25% penetration level.

Therefore, considering only the factor of fitting ability, the

linearized Fisher-Pry model would probably provide the analyst with the

worst results at 25% penetration level.

Table 11 contains the analysis of variance results when Tukey's test was applied to the relative fitting errors of growth models at 50% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

TABLE 11. ANOVA of fitting error at 50% level

source of variation	d.f.	sum of squares	mean square
model	5	45.65470536	
error	126	122.68513946	0.97369158
total	131	168.33984482	

$$Q = 4.093$$
$$s = (0.97369158/22)^{1/2} = 0.210369$$
$$D = Q \cdot s = 0.86104$$

Table 12 contains the differences between sample means of the six growth models which are compared to Tukey's D of 0.86104.

The H_4 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other growth models. This result indicates that the linearized Fisher-Pry model appears to have a larger fitting error than the other growth models at 50% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry model would probably provide the analyst with the worst results at 50% penetration level.

TABLE 12. Fitting differences between models at 50% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(2.17)	–	1.3461*	1.1311*	1.5496*	1.6743*	1.7196*
LGZ(0.82)		–	0.2150	0.2035	0.3282	0.3736
LWB(1.04)			–	0.4186	0.5432	0.5886
PL(0.62)				–	0.1247	0.1700
GZ(0.49)					–	0.0453
WB(0.45)						–

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve
     * = indicates significant difference exists between models

Table 13 contains the analysis of variance results when Tukey's
test was applied to the relative fitting errors of growth models at 75%
penetration level. Six treatments and 126 degrees of freedom are used
to determine Q.

TABLE 13. ANOVA of fitting error at 75% level

source of variation	d.f.	sum of squares	mean square
model	5	117.12870580	
error	126	296.94189558	2.35668171
total	131	414.07060138	

$Q = 4.093$
$s = (2.35668171/22)^{1/2} = 0.327295$
$D = Q \cdot s = 1.3396$

Table 14 contains the differences between sample means of the six growth models which are compared to Tukey's D of 1.3396.

TABLE 14. Fitting differences between models at 75% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(3.75)	-	2.2487*	1.7194*	2.2822*	2.7970*	2.6927*
LGZ(1.50)		-	0.5293	0.0335	0.5483	0.4440
LWB(2.03)			-	0.5629	1.0776	0.9733
PL(1.47)				-	0.5148	0.4104
GZ(0.95)					-	0.1043
WB(1.05)						-

Key: LFP = linearized Fisher-Pry model
    LGZ = linearized Gompertz growth curve
    LWB = linearized Weibull growth curve
    PL = Pearl growth curve
    GZ = Gompertz growth curve
    WB = Weibull growth curve
     * = indicates significant difference exists between models

The H_4 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other growth models. This result indicates that the linearized Fisher-Pry model appears to have a larger fitting error than the other growth models at 75% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry model would probably provide the analyst with the worst results at 75% penetration level.

Table 15 contains the analysis of variance results when Tukey's test was applied to the relative fitting errors of growth models at 100% penetration level. Six treatments and 126 degrees of freedom are used to determine Q.

TABLE 15. ANOVA of fitting error at 100% level

source of variation	d.f.	sum of squares	mean square
model	5	516.62353322	
error	126	1219.77233965	9.68073285
total	131	1736.39587287	

$$Q = 4.093$$
$$s = (9.68073285/22)^{1/2} = 0.663350$$
$$D = Q \cdot s = 2.715$$

Table 16 contains the difference between sample means of the six growth models which are compared to Tukey's D of 2.715.

The H_1 hypothesis is rejected at the 5% of significance, indicating that a statistical difference exists between the linearized Fisher-Pry model and the other nonlinearized growth models and between the linearized Weibull growth curve and the other nonlinearized growth models. This result indicates that the linearized Fisher-Pry model and the linearized Weibull curve appear to have a larger fitting error than the other nonlinearized growth models at 100% penetration level. Therefore, considering only the factor of fitting ability, the linearized Fisher-Pry and the linearized Weibull curve would probably provide the analyst with worse results at 100% penetration levels.

TABLE 16. Fitting differences between models at 100% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(6.23)	—	2.0901	0.3769	4.1826*	4.7027*	4.7535*
LGZ(4.14)		—	1.7132	2.0925	2.6125	2.6634
LWB(5.86)			—	3.8057*	4.3258*	4.3766*
PL(2.05)				—	0.5200	0.5709
GZ(1.53)					—	0.0508
WB(1.48)						—

Key: LFP = linearized Fisher-Pry model
    LGZ = linearized Gompertz growth curve
    LWB = linearized Weibull growth curve
    PL = Pearl growth curve
    GZ = Gompertz growth curve
    WB = Weibull growth curve
    * = indicates significant difference exists between models

Table 17 shows that the linearized versus the nonlinearized growth models of fitting ability for each model, at each penetration level. The bracketed number after the name of model is the mean for the twenty-two case of the mean estimate error using each model.

This result indicates that a difference in the fitting ability of the models does exist at the 5% level of significance. Nonlinearized growth models appear to provide a significantly lower fitting error than do the linear version of growth models. Therefore, if statistical fitting is the criterion used to select a forecasting model, nonlinearized growth models would probably provide the analyst with better results.

TABLE 17. Linearized vs. nonlinearized fitting error

level	model		
5%	LFP(0.0148) PL(0.0060) F = 4.83 P>F = 0.0393*	LGZ(0.0086) GZ(0.0059) F = 7.23 P>F = 0.0137*	LWB(0.0072) WB(0.0051) F = 6.14 P>F = 0.0218*
10%	LFP(0.0758) PL(0.0304) F = 6.15 P>F = 0.0217*	LGZ(0.0437)· GZ(0.0282) F = 8.89 P>F = 0.0071*	LWB(0.0422) WB(0.0246) F = 6.79 P>F = 0.0165*
25%	LFP(0.6192) PL(0.1875) F = 9.26 P>F = 0.0062*	LGZ(0.2552) GZ(0.1580) F = 10.35 P>F = 0.0041*	LWB(0.2446) WB(0.1520) F = 12.86 P>F = 0.0017*
50%	LFP(2.1681) PL(0.6185) F = 18.49 P>F = 0.0003*	LGZ(0.8220) GZ(0.4938) F = 12.80 P>F = 0.0018*	LWB(1.0370) WB(0.4484) F = 17.28 P>F = 0.0004*
75%	LFP(3.7475) PL(1.4652) F = 20.63 P>F = 0.0002*	LGZ(1.4988) GZ(0.9505) F = 14.53 P>F = 0.0010*	LWB(2.0280) WB(1.0548) F = 17.77 P>F = 0.0004*
100%	LFP(6.2320) PL(2.0494) F = 24.42 P>F = 0.0001*	LGZ(4.1419) GZ(1.5294) F = 9.63 P>F = 0.0054*	LWB(5.8552) WB(1.4785) F = 29.00 P>F = 0.0001*

Key: P>F = probability of a higher value of F-value
     * = indicates significantly different with 5% confidence

Results of Kruskal-Wallis test

A nonparametric approach is applied to the same data in order to see if results would change with the relaxing of the analysis of

variance assumptions that observations are selected from a normally
distributed population. Nonparametric tests are distribution-free
tests and do not rely on the assumption of normality. Since the sample
size for all six groups of growth models exceeds five, the chi-square
approximation is used for the Kruskal-Wallis test. The approximation
appears to be fairly good when sample sizes exceed five [79]. Table 18
contains the results of the Kruskal-Wallis test for the growth models
at different penetration levels.

TABLE 18. Results of Kruskal-Wallis test (fitting ability)

penetration level	test value	probability
5%	8.55	0.1282
10%	11.83	0.0373*
25%	15.84	0.0073*
50%	22.57	0.0004*
75%	22.13	0.0005*
100%	44.87	0.0001*

Key: * = indicates significantly different with 5% confidence

The results of Table 18 suggest that no significant difference is
detected among the fitting errors of the six growth models at 5%
penetration level; however, a significant difference seems to exist
among the fitting errors of the six growth models at 10%, 25%, 50%,
75%, and 100% penetration level. These results are consistent with
those produced by performing Tukey's test on the data.

Results for Comparison of the Forecasting Ability

Results of Tukey test

The comparison among the forecasting errors of growth models is performed using Tukey's test. Table 19 contains the differences between forecast errors at 5% penetration level which are compared to Tukey's D of 88.714. The bracketed number after the name of model is the mean for the twenty-two cases of the average squared forecast error using each growth model as defined in equation (65).

TABLE 19. Forecast differences between models at 5% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(54.26)	—	5.78	83.91	20.85	2.51	81.71
LGZ(48.47)		—	89.70*	26.63	3.27	87.49
LWB(138.2)			—	63.07	86.42	2.20
PL(75.10)				—	23.36	60.80
GZ(51.74)					—	84.22
WB(136.0)						—

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve
      * = indicates significant difference exists between models

The H_5 (there exists no significant difference among the the forecasting errors of growth models) hypothesis is rejected at the 5%

level of significance, indicating that the linearized Weibull growth
curve appears to have a larger forecasting error than the linearized
Gompertz growth curve at 5% penetration level.

Table 20 contains the differences between forecast errors of the
six growth models performed at 10% penetration level.

TABLE 20. Forecast differences between models at 10% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(56.67)	–	20.21	22.90	23.76	23.70	27.73
LGZ(36.47)		–	43.10	43.97	3.49	47.94
LWB(79.57)			–	0.86	46.59	4.84
PL(80.43)				–	47.46	3.97
GZ(32.97)					–	51.43
WB(84.41)						–

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

As can be seen from Table 20, none of the difference exceeds the
calculated Tukey's D of 75.31; therefore, H_5 hypothesis cannot be
rejected at the 5% significance level. There is not sufficient
evidence to show that a true difference exists between any of the
growth models at 10% penetration level.

Table 21 contains the differences between forecast errors at 25%
penetration level which are compared compared to Tukey's D of 31.265.

TABLE 21. Forecast differences between models at 25% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(33.37)	-	3.42	21.60	10.93	5.63	16.16
LGZ(29.95)		-	25.02	7.51	2.21	19.58
LWB(54.96)			-	32.52*	27.23	5.44
PL(22.44)				-	5.30	27.08
GZ(27.74)					-	21.79
WB(49.52)						-

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve
      * = indicates significant difference exists between models

The H_5 hypothesis is rejected at the 5% of significance, indicating that the linearized Weibull growth curve appears to have a larger forecasting error than the Pearl growth curve at 25% penetration level.

Table 22 contains the difference between forecast errors of the six growth models performed at 50% penetration level.

As can be seen from Table 22, none of the difference exceeds the calculated Tukey's D of 13.253; therefore, H_5 hypothesis cannot be rejected at the 5% significance level. There is not sufficient evidence to show that a true difference exists between any of the growth models at 50% penetration level.

TABLE 22. Forecast differences between models at 50% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(18.79)	–	2.821	0.772	8.643	8.548	3.391
LGZ(15.97)		–	3.592	5.823	5.727	0.570
LWB(19.56)	.		–	9.415	9.320	4.163
PL(10.15)				–	0.095	5.252
GZ(10.24)					–	5.157
WB(15.40)				.		–

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

Table 23 contains the difference between forecast errors of the six growth models performed at 75% penetration level.

As can be seen from Table 23, none of the difference exceeds the calculated Tukey's D of 7.0837; therefore, H_5 hypothesis cannot be rejected at the 5% significance level. There is not sufficient evidence to show that a true difference exists between any of the growth models at 75% penetration level.

Table 24 shows that the linearized versus the nonlinearized growth models of forecasting ability for each model, at each penetration level. The bracketed number after the name of model is the mean for the twenty-two case of the average squared forecast error using each model.

TABLE 23. Forecast differences between models at 75% level

	LFP	LGZ	LWB	PL	GZ	WB
LFP(8.30)	—	0.716	1.481	3.550	2.565	3.150
LGZ(9.02)		—	0.765	4.266	3.280	3.865
LWB(9.78)			—	5.031	4.046	4.631
PL(4.75)				—	0.985	0.400
GZ(5.74)					—	0.585
WB(5.15)						—

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

The results of Table 24 suggest that no significant difference appears to be detected among the forecasting errors of the growth model at lower penetration levels; however, nonlinearized growth models improve the forecasting ability of most of models especially at higher penetration levels.

Results of Kruskal-Wallis test

A nonparametric approach is applied to the same data in order to see if results would change with the relaxing of the analysis of variance assumptions. Table 25 contains the results of the Kruskal-Wallis test for the growth models at different penetration levels.

The results of Table 25 suggest that no significant different appears to be detected among the forecasting errors of the six growth

TABLE 24. Linearized vs. nonlinearized forecasting error

level	model		
5%	LFP(54.26) PL(75.10) F = 0.87 P>F = 0.3624	LGZ(48.47) GZ(51.74) F = 0.15 P>F = 0.7069	LWB(138.17) WB(135.96) F = 0.08 P>F = 0.7834
10%	LFP(56.67) PL(80.43) F = 0.62 P>F = 0.4391	LGZ(36.47) GZ(32.97) F = 0.21 P>F = 0.6551	LWB(79.57) WB(84.41) F = 0.26 P>F = 0.6124
25%	LFP(33.37) PL(22.44) F = 7.29 P>F = 0.0134*	LGZ(29.95) GZ(27.74) F = 0.20 P>F = 0.6564	LWB(54.96) WB(49.52) F = 0.39 P>F = 0.5407
50%	LFP(18.79) PL(10.15) F = 12.11 P>F = 0.0022*	LGZ(15.97) GZ(10.24) F = 3.48 P>F = 0.0762	LWB(19.56) WB(15.40) F = 3.31 P>F = 0.0830
75%	LFP(8.300) PL(4.750) F = 9.85 P>F = 0.0050*	LGZ(9.020) GZ(5.740) F = 4.69 P>F = 0.0420*	LWB(9.780) WB(5.150) F = 11.51 P>F = 0.0027*

Key: P>F = probability of a higher value of F-value
* = indicates significantly different with 5% confidence

models at 50% and 75% penetration level; however, a significant difference seems to exist among the forecasting errors of the six growth models at 5%, 10%, and 25% penetration level. These results are consistent with those produced by performing Tukey's test on the data, except at 10% penetration level.

TABLE 25. Results of Kruskal-Wallis test (forecasting ability)

penetration level	test value	probability
5%	12.22	0.0318*
10%	13.08	0.0226*
25%	14.71	0.0117*
50%	8.97	0.1102
75%	5.36	0.3731

Key: * = indicates significantly different with 5% confidence

The general result that must be noted is the performance of the version of Weibull curves. For fitting ability, the version of Weibull curves and Gompertz curves are statistically better than the logistic curves; however, the Weibull curves are statistically the worst in forecasting ability. For this reason, only the statistically better models were used, namely the logistic curves and the Gompertz curves, for forecasting.

CHAPTER VIII. SIMULATED GROWTH CURVES

The empirical results of using six technological growth models and
the poor performance of the Weibull curves for forecasting ability have
been tested in Chapter VII. The dilemma lies in choosing an
appropriate model. Some analysts favor the use of a common form of the
nonlinearized models, such as the Pearl growth curve or the Gompertz
curve; still others prefer to employ transformations of the models,
such as the linearized Fisher-Pry model or the linearized Gompertz
growth curve.

Rather than consistently utilizing one version of a growth model
for all technological phenomena, an approach is suggested to develop
diagnostics which distinguish between the logistic growth curves and
the version of Gompertz curves as the general form of a growth curve,
and then to develop further criteria which will determine the preferred
transformation of growth curve in each version.

Simulation of Growth Models

Once the models have been specified as done in Chapter VII,
simulations are run to create several data series with known population
growth models. Simulations were developed in SAS using its random
number generator to create the stochastic element at each model.

Table 26 contains the summarized results of the factor levels for
the standard errors of error terms, the parameter terms, and the length
of simulated data using the four growth models, calculated from the

twenty-two historical growth cases. This table is derived from the individual case results contained in Appendix K, Appendix L, and Appendix M.

TABLE 26. Factor levels for simulated models

model	standard error	scale parameter	shape parameter	length
PL	0.04184	40.774	0.1811	40
GZ	0.03612	7.532	0.1254	40
LFP	0.6326	-3.848	0.1924	40
LGZ	0.3358	-1.782	0.1209	40

Key: LFP = linearized Fisher-Pry model
    LGZ = linearized Gompertz growth curve
    PL = Pearl growth curve
    GZ = Gompertz growth curve

All four models are used to the simulated data for fitting and forecasting, utilizing the Marquardt nonlinear algorithm provided in SAS programs for the Pearl curve and the Gompertz curve; the ordinary least squares method is used for the linearized Fisher-Pry model and the linearized Gompertz curve. Ten replicates were run for each penetration level. This results in nine hundred and sixty runs for test of fitting ability and eight hundred runs for test of forecasting ability.

Diagnostics of Growth Models

With knowledge of the underlying population model, the techniques for model selection could be created and then tested for accuracy. Two key aspects need to be studied to determine the appropriate model: the general form of the trend (degree of skew) and the underlying error structure (shape of variance function over time). In order to specify the appropriate choice of both characteristics, diagnostics are employed to aid in choosing the appropriate model.

Skew of models

Two diagnostic procedures are proposed to determine whether data conform better to the logistic versions or the Gompertz versions. The first is a graphical procedure which examines plots of the data and the second is a post-fitting diagnostic.

Graphical procedure   For the graphical procedure, suitable transformations are needed which will produce straight line plots of the data when the underlying model is correct. The appropriate transformation for the logistic versions is

$$L(t) = \ln \left(\frac{Y(t)}{1 - Y(t)}\right) \tag{66}$$

whereas for the Gompertz versions,

$$G(t) = -\ln\{-\ln[Y(t)]\} \tag{67}$$

These transformations are applied to each set of simulated data. If the data are the logistic versions in nature, equation (66) results in a linear plot of L(t) against time t, whereas equation (67) produces a slightly curved plot which is convex to the origin (the plot becomes steeper as t increase). Figure 5 and Figure 6 illustrate these phenomena.
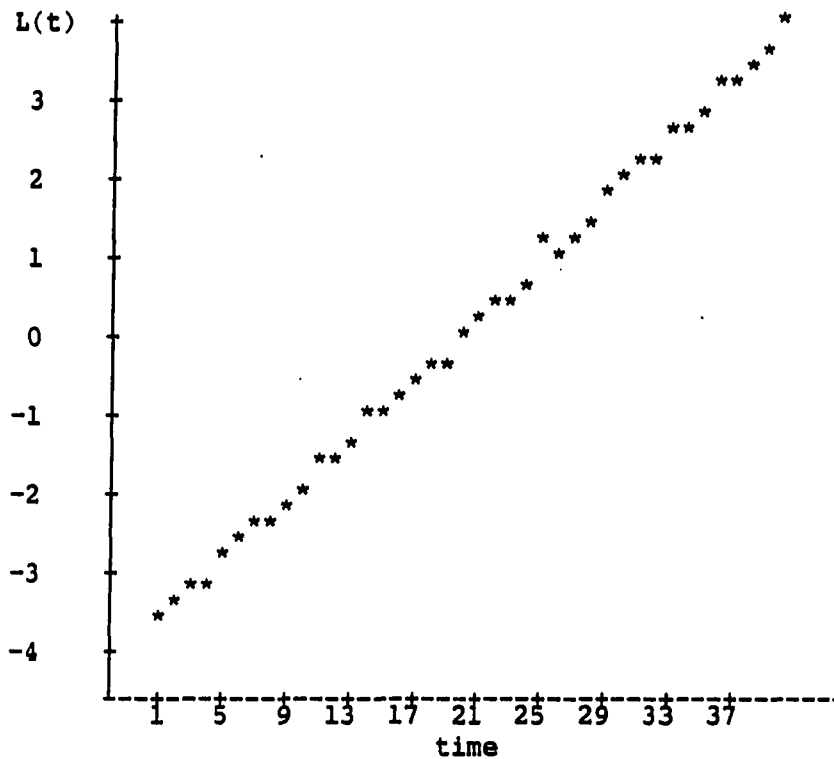
FIGURE 5. Linear logistic plot of logistic simulation

If the data are based on the Gompertz models, equation (66) produces a slightly curved plot which is concave to the origin (i.e.,
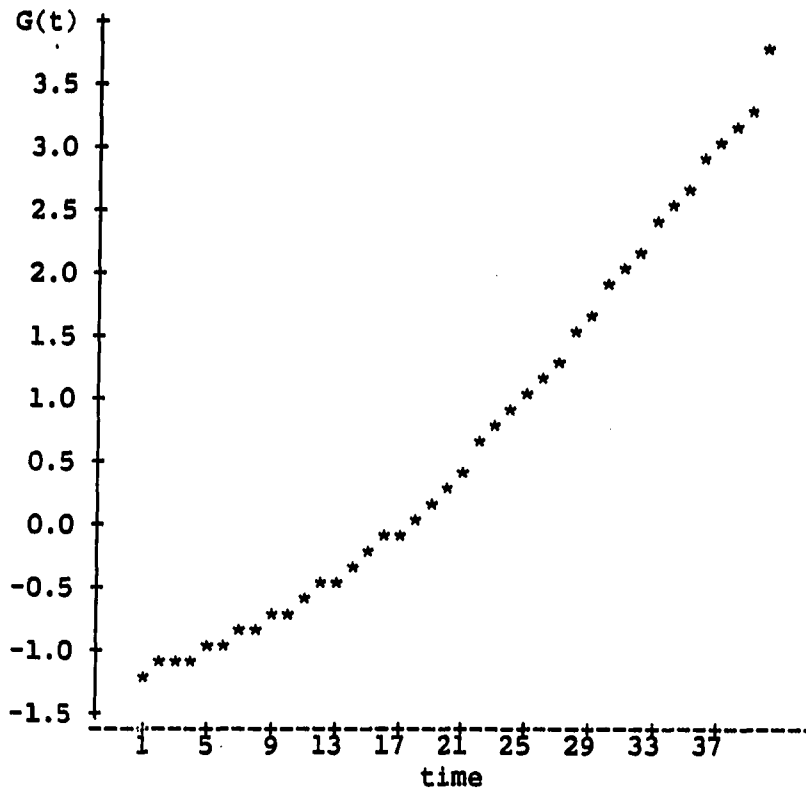
FIGURE 6. Linear Gompertz plot of logistic simulation

the plot flattens out as t increases), whereas equation (67) provides a linear plot. Figure 7 and Figure 8 demonstrate these phenomena.

Post-fitting diagnostic   A modified mean square error (i.e., mean estimate error) is used as a post-fitting diagnostic for model selections. If the smallest mean estimate error is for the Pearl or the linearized Fisher-Pry model, the logistic version is chosen. If the Gompertz or the linearized Gompertz curve yields the smallest mean estimate error, then the Gompertz version is chosen.
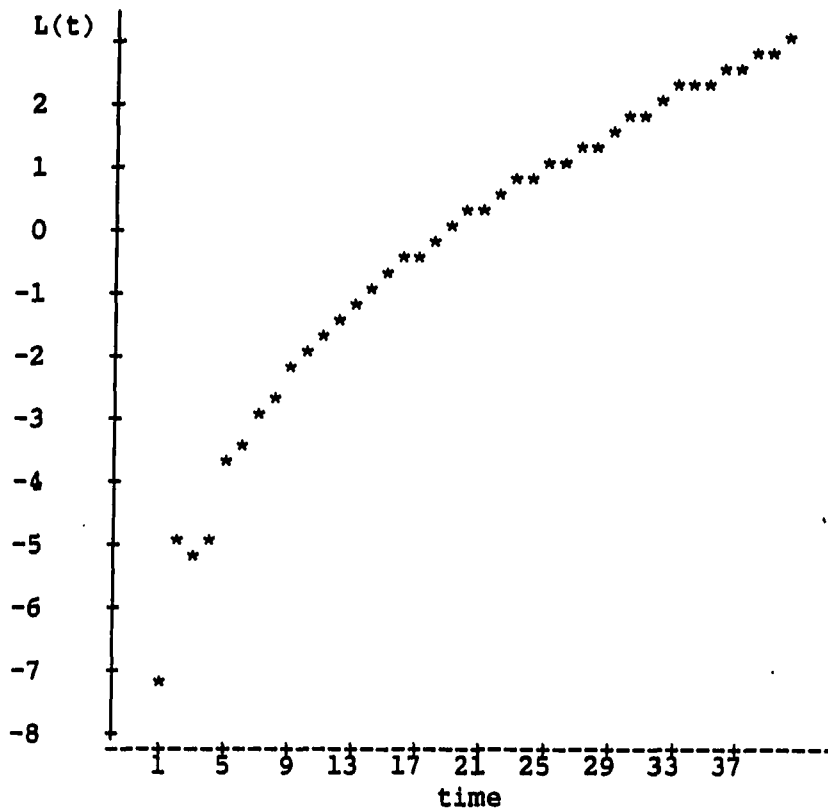
FIGURE 7. Linear logistic plot of Gompertz simulation

Error structures of models

In order to determine the appropriate form of the error structure, the mean estimate error was supposed to select the best form of the error structure. The smallest mean estimate error for the four fitted models may be determine both the best model and the best transformation. But the mean estimate error proved to be inadequate, as will be shown in the next section. To compensate, another pre-fitting diagnostic was devised to permit the analyst to choose the

```
G(t)
 3.0
 2.5
 2.0
 1.5
 1.0
 0.5
 0.0
-0.5
-1.0
-1.5
-2.0
        1   5   9  13  17  21  25  29  33  37
                        time
```
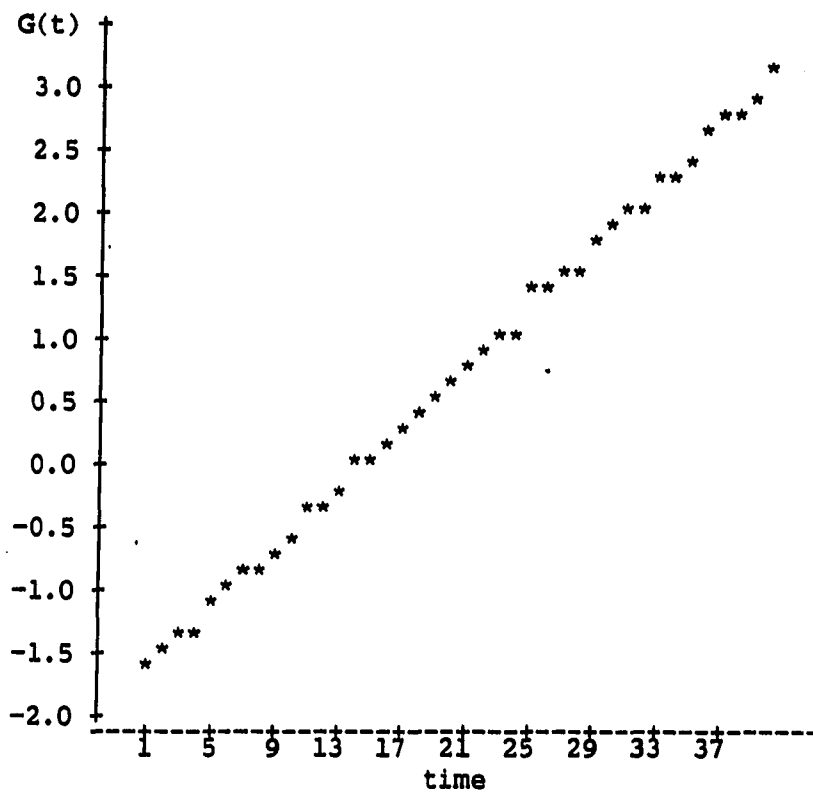
FIGURE 8. Linear Gompertz plot of Gompertz simulation

appropriate transformation of a growth curve. As shown in Chapter VI, for the Pearl curve or the Gompertz curve, the errors have zero mean and a constant variance over time. Transformed models, however, like the linearized Fisher-Pry model or the linearized Gompertz curve has increasing variance from t = 0 to the inflection point.

A pre-fitting diagnostic is devised to determine whether the variance of error terms is constant or increasing in structure. A time series of pseudo-residuals is used by subtracting the three point moving average from each Y(t),

$$R(t) = Y(t) - \frac{1}{3} [Y(t-1) + Y(t) + Y(t+1)] \qquad (68)$$

These pseudo-residuals are then plotted against time to ascertain whether the variation of the data points is constant or increasing over time. Such a plot could allow for the visual determination of constant versus increasing variance over time.

An example of such a residual plot of the Pearl growth curve has been provided in Figure 9 and the linearized Fisher-Pry model is plotted in Figure 10.
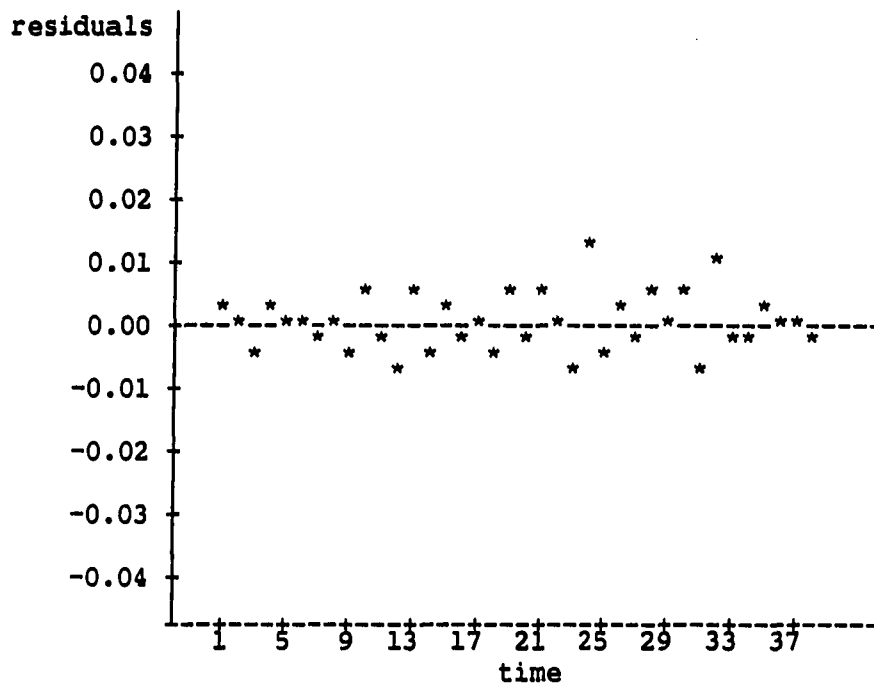
FIGURE 9. Residuals plot of Pearl simulation
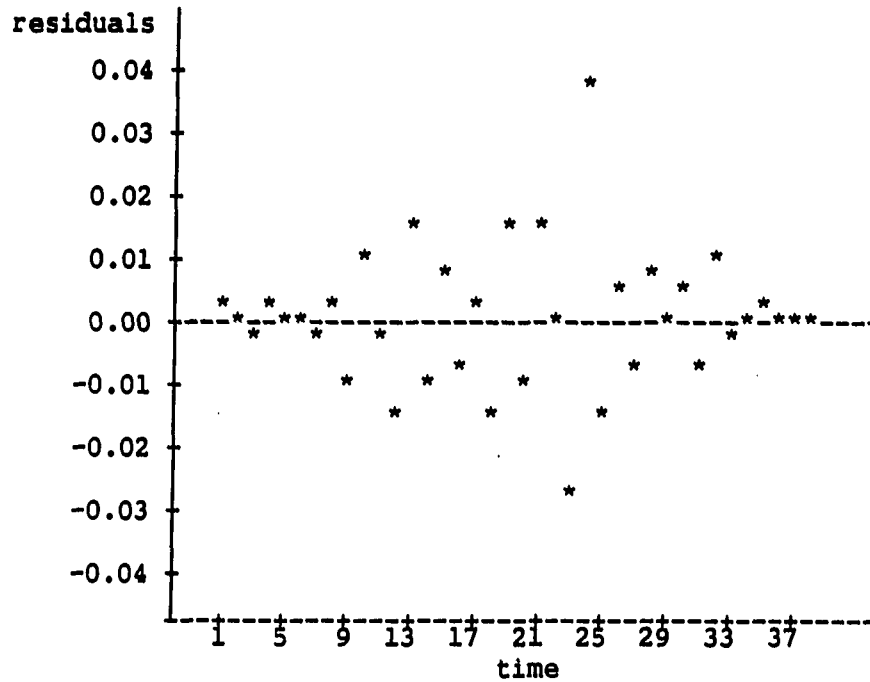
FIGURE 10. Residuals plot of linearized Fisher-Pry simulation

Figure 11 and Figure 12 demonstrate a typical residual plot for the Gompertz and the linearized Gompertz growth curve respectively.

Results of Diagnostic Tests

The diagnostic tests to determine the skew and the error structure appropriate for a given set of simulated data was performed and tested the effectiveness of these criteria.

Results of skew diagnostic

As described previously, two techniques were employed to determine the general form of the growth curve.
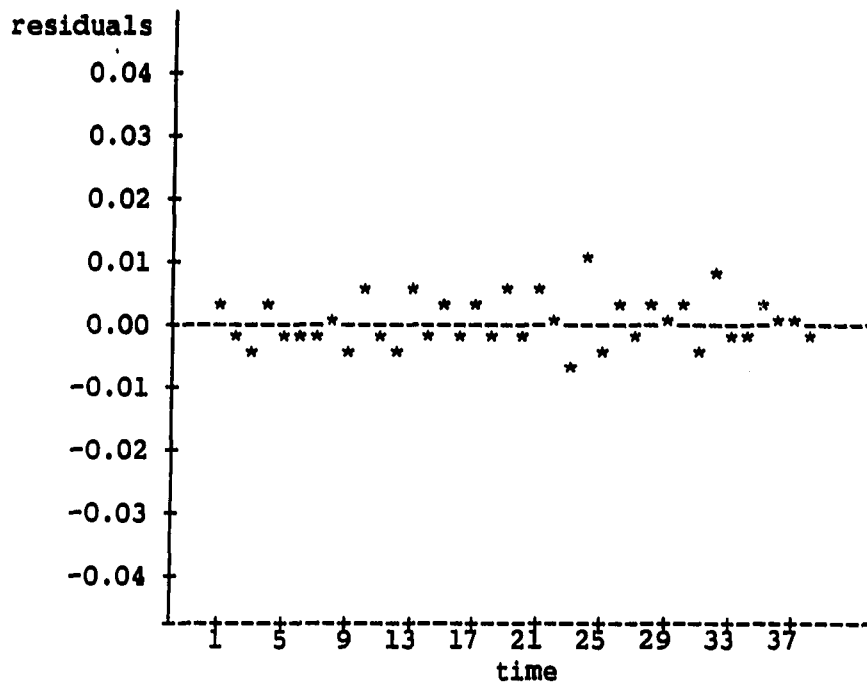
FIGURE 11. Residuals plot of Gompertz simulation

Results of graphical procedure The graphical procedure is performed as to which transformed data plot yields the straighter line. Two hundred forty different pairs of graphs (four models with six different penetration levels, repeated ten times) were judged independently by two experts. Their results, broken down in the number of correct decisions of the logistic versions or the Gompertz versions, are summarized in Table 27.

Even though the experts used a graphical plot, as shown in Table 27, it appears that this pre-fit diagnostic would be appropriate.
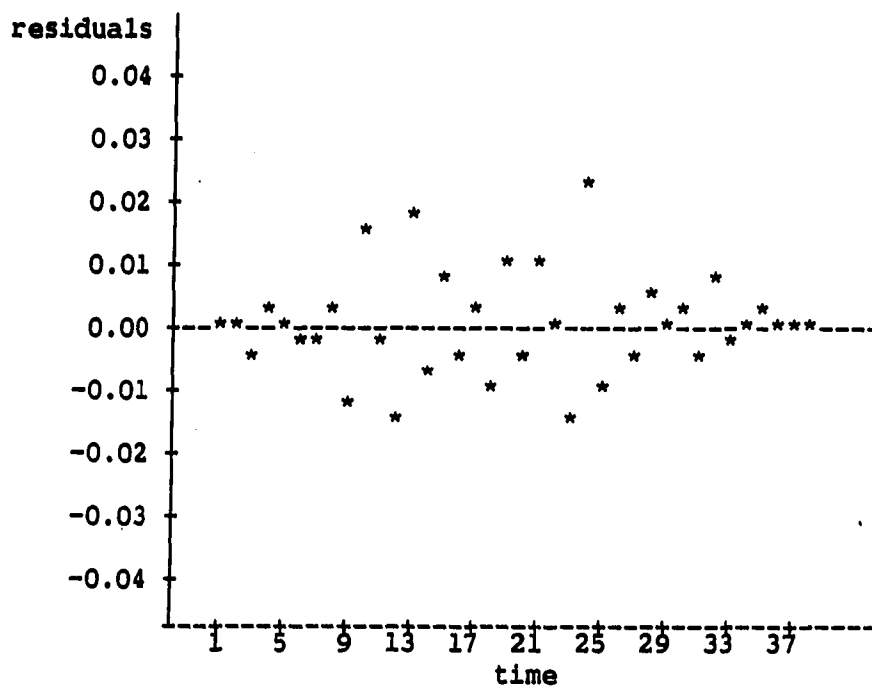
FIGURE 12. Residuals plot of linearized Gompertz simulation

Results of post-fitting diagnostic   The results of the post-
fitting diagnostic of choosing the model with the smallest mean
estimate error are included in Table 28.

In the general model determination phase, it appears that either
the graphical diagnostic (i.e., choosing the straighter of the two
lines) or the post-fitting diagnostic (i.e., choosing logistic or
Gompertz according to the smaller mean estimate error) would be
appropriate.

TABLE 27. Decisions of general form by experts

penetration level		5%		10%		25%		50%		75%		100%	
simulated model	decision	expert											
		1	2	1	2	1	2	1	2	1	2	1	2
PL	Logistic	6	7	8	7	9	9	9	9	9	9	10	10
	Gompertz	4	3	2	3	1	1	1	1	1	1	0	0
LFP	Logistic	6	7	9	8	9	10 ·	10	10	10	10	10	10
	Gompertz	4	3	1	2	1	0	0	0	0	0	0	0
GZ	Logistic	1	2	1	1	1	1	0	0	0	0	0	0
	Gompertz	9	8	9	9	9	9	10	10	9	9	10	10
LGZ	Logistic	1	1	0	0	1	0	0	0	1	1	0	0
	Gompertz	9	9	10	10	9	10	10	10	9	9	10	10
total number missed		10	9	4	6	4	2	1	1	2	2	0	0

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

     correct decisions/total: expert 1: 219/240 = 0.913
                              expert 2: 220/240 = 0.916

TABLE 28. Determination of general form by mean estimate error

	simulated models			
	PL	LFP	GZ	LGZ
correct	57	57	60	60
incorrect	3	3	0	0
total	60	60	60	60

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

Results of error structure diagnostic

It was originally thought that the smallest mean estimate error would provide the means for determining the underlying error structure of the model. However, as shown in Table 29, the mean estimate error of the Pearl growth curve was smaller than the linearized Fish-Pry models, and the mean estimate error of the Gompertz curve was smaller than the linearized Gompertz curve

TABLE 29. Error structure chosen by mean estimate error

smallest mean	simulated models			
estimate error	PL	LFP	GZ	LGZ
PL	56	51	0	0
LFP	1	6	0	0
GZ	2	2	60	58
LGZ	1	1	0	2
total missed	4	54	0	58

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

Note that, when fitting the linearized Fisher-Pry model or the linearized Gompertz curve on their own simulations, the mean estimate error criterion could not select the correct model. Since the mean estimate error criterion tends to incorrectly identify the transformed models, a pseudo-residuals plot is provided for determining the underlying error structure of the model.

To test the diagnostic of the pseudo-residuals, another expert was chosen to separate eighty plots of the simulated data (four models with two different penetration levels, repeated ten times each) into two groups: constant variance or increasing variance. Table 30 provides the results of test.

TABLE 30. Error structure chosen by residuals plot

| | simulated models | | | |
	PL	LFP	GZ	LGZ
constant variance	19	2	18	1
increasing variance	1	18	2	19
total missed	1	2	2	1

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

Note that only six plots were classified incorrectly. Considering the high degree of inaccuracy for choosing the appropriate error structure when using the mean estimate error criterion, the diagnostic of pseudo-residuals plot is more appropriate for specifying the underlying error structure.

Selection Procedure for Growth Models

For choosing the general form of growth models, both the graphical diagnostic and the mean estimate error criterion work equally well in selection. For error structure, the diagnostic of pseudo-residuals plot is more accurate means of selection growth model than the mean estimate error criterion.

However, before the final procedure is developed to select the appropriate growth model, the forecasting ability was compared. Table 31 provides the summarized decision of the smallest mean forecast error for all two hundred simulated data (four models with five different penetration levels, repeated ten times each).

TABLE 31. Smallest mean forecast error for simulated data

smallest mean forecast error	simulated models			
	PL	LFP	GZ	LGZ
PL	34	25	0	0
LFP	16	25	0	0
GZ	0	0	40	26
LGZ	0	0	10	24
total	50	50	50	50

Key: LFP = linearized Fisher-Pry model
LGZ = linearized Gompertz growth curve
PL = Pearl growth curve
GZ = Gompertz growth curve

This table illustrates that knowledge of the general form of the model does improve the accuracy of forecasting, but knowledge of the error structure does not prove to be as helpful. As shown in Table 31, for the linearized Fisher-Pry model, the Pearl growth model performs better twenty-five out of fifty times; whereas, for the linearized Gompertz model, the Gompertz model provides a better forecast twenty-six out of fifty times. The skew of the model seems to be more important than the error structure underlying that model for forecasting ability.

This result indicates two possible methods for selecting the growth curve. If the analyst wishes to fit properly the sample data with the model that represents the underlying population, then the following procedures could be appropriate.

1. Plot the linear transformation of data, assuming both the logistic model or the version of Gompertz curves, and choose the straighter of the two plots. The straighter line determines the general form of the model.

   or

   Find a mean estimate error. If the smallest mean estimate error is for the Pearl or the linearized Fisher-Pry model, the logistic version is chosen. If the Gompertz or the linearized Gompertz curve yields the smallest mean estimate error, then the Gompertz version is chosen.

2. Once the general form of model has been chosen, plot the pseudo-residuals of the observed data from their three point moving averages. If the plots display a constant variance over time, choose the original form of model (i.e., the Pearl curve or the Gompertz curve). If the plots display an increasing variance over time, choose the transformed model (i.e., the linearized Fisher-Pry model or the linearized Gompertz curve).

However, if the analyst is more concerned with the forecasting ability, rather than the accuracy of the fitting ability, then following procedure could be appropriate.

1. Plot the linear transformation of data, assuming both the logistic model or the version of Gompertz curves, and choose the straighter of the two plots. The straighter line determines the general form of the model.

or

Find a mean estimate error. If the smallest mean estimate error is for the Pearl or the linearized Fisher-Pry model, the logistic version is chosen. If the Gompertz or the linearized Gompertz curve yields the smallest mean estimate error, then the Gompertz version is chosen.

2. If the logistic form of the model has been chosen, the Pearl growth curve is utilized to forecast the data. If the

Gompertz version of the model has been chosen, the Gompertz growth is utilized to forecast the data.

CHAPTER IX. APPLICATION IN LIFE ANALYSIS

In the previous chapter, procedures for model selection were examined by employing simulated time series data. Since the procedures have been established using simulated data, it was considered worthwhile to test the procedures on real time series data.

The resultant model, as selected through the procedures, is then fitted using discounted least squares and employed to make forecasts on real time series data. A proposal for incorporating growth models into the traditional life analysis framework is given and demonstrated with an example.

Application of Model Selection

In choosing the appropriate growth model, the pre-fit diagnostics were used to the selection of general form (by choosing the transformed plot that appears to represent the straighter line) and error structure (by examining the plot of the residuals for constant or increasing variance). This implies that the resultant model will be one of the four growth curves: Pearl, linearized Fisher-Pry, Gompertz or linearized Gompertz. Minimum mean square errors will be employed only to confirm the choice of the general form of the growth model.

Once the appropriate form of the model has been specified, discounted least squares is employed to estimate the parameters of that model and to forecast that model ahead for several periods. The last five data points are set aside in order to test the forecasting ability

of the model with five sets of parameters which are derived from five levels of discounting (i.e., w=1.00, 0.95, 0.90, 0.80 and 0.70). The model is determined to be the best if it provides the minimum forecast errors with respect to that actual data points.

Once the best discount factor is chosen, the final model would be the result of fitting the entire data set using that discount factor. The substitution of electronic (stored program control or SPC) for electromechnical switching in the telephone industry was chosen to test the model selection procedures. Data from ten Bell Operating Companies (BOCs) were tested.

Case study of Company 9

The data series, which measures the percentage of substitution of electronic for electromechnical switching in the telephone Company 9, has nineteen observations. Table 32 provides the actual data which date from 1967 to 1985. The data range from 0.07 percent to 52.8 percent, which indicates that the technology is relatively young in its growth pattern.

The first stage of the analysis is the determination of general form. The actual time series is fitted onto the linear logistic and the linear Gompertz equations. The plots of the transformations are provided in Figure 13 (linear logistic plot) and Figure 14 (linear Gompertz plot).

TABLE 32. Percentage of electronic switching of Company 9

year	% electronic	year	% electronic
1967	0.0653	1977	20.0954
1968	0.3790	1978	26.2527
1969	0.4426	1979	30.4281
1970	1.9997	1980	32.3323
1971	2.9561	1981	35.5104
1972	5.9636	1982	37.7081
1973	11.2405	1983	41.1211
1974	14.5386	1984	47.4859
1975	16.7147	1985	52.7996
1976	19.0572		
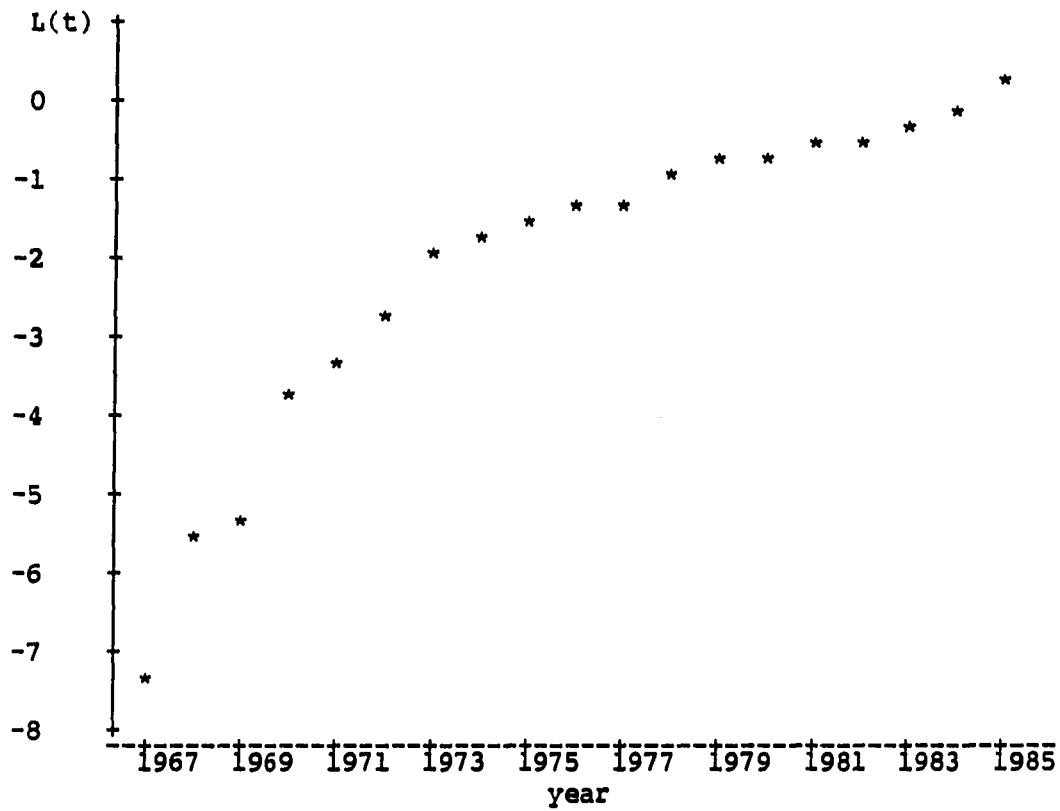
FIGURE 13. Linear logistic plot for Company 9
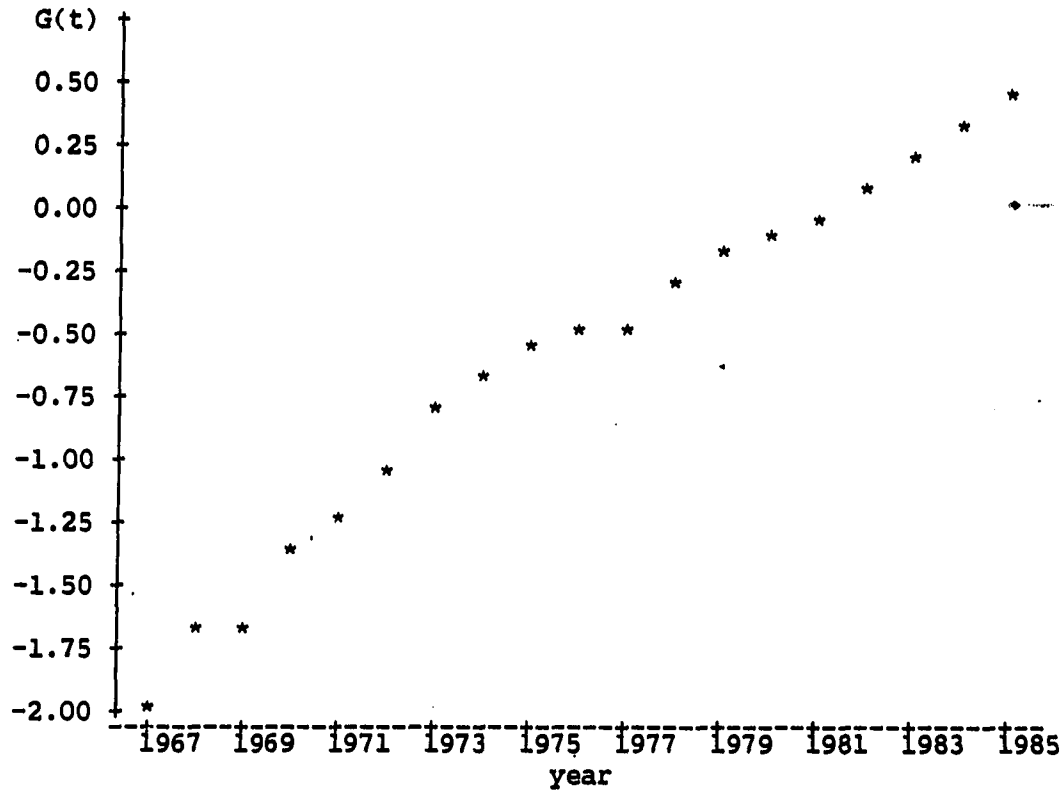
FIGURE 14. Linear Gompertz plot for Company 9

A visual inspection of the data indicates that the Gompertz
transformation results in a straighter line than does the logistic
transformation. Therefore, the Gompertz version is chosen as the
appropriate general form of the model.

For the next stage of the analysis, the plot of pseudo-residuals
is provided in Figure 15. The plot of the residuals indicates a
constant variance pattern. This implies that the Gompertz model
provides the best estimate of the underlying population. The Gompertz
model is chosen as representative of the population that derives the
data of the Company 9.
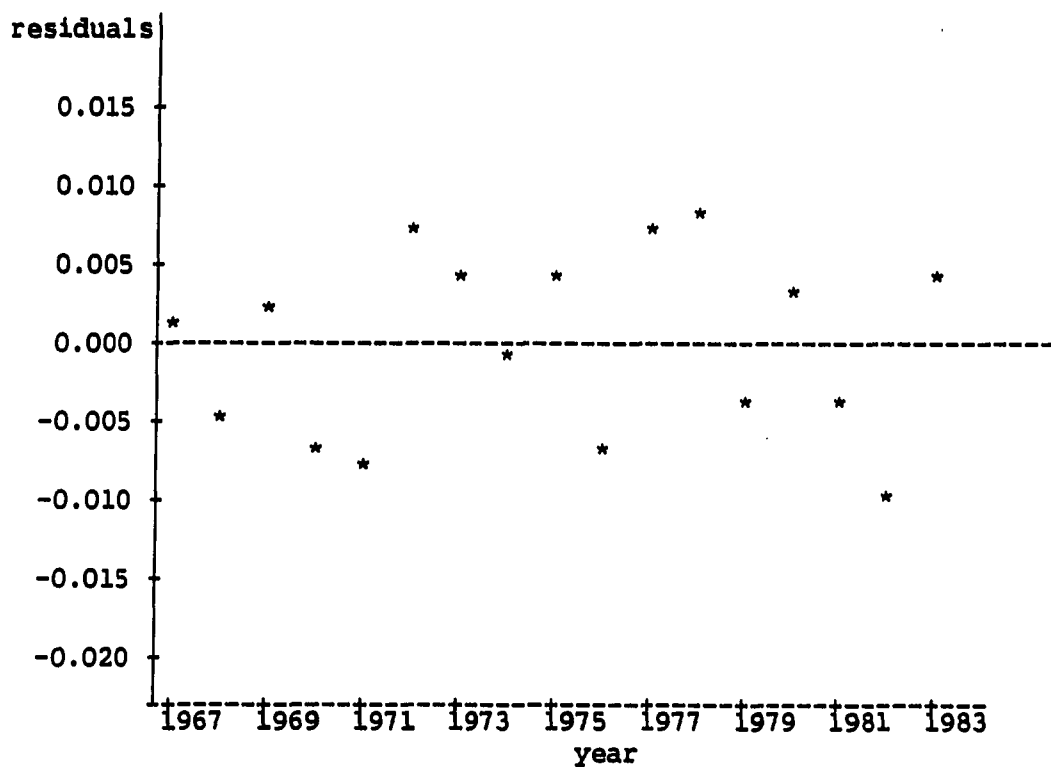
FIGURE 15. Residual plot of Company 9

The mean estimated errors for each of the four growth models as applied to the Company 9 time series data are stated in Table 33. For the Company 9 data, the Gompertz model has the lowest mean estimate error, which confirms the choice made by plots of the linear transformations.

Figure 16 provides the plot of the actual time series data of the Company 9 against the four fitted time series.

Using the nonlinear program that was employed in previous Chapter VIII, the Gompertz model is fitted onto the data with the five discount

TABLE 33. Mean estimate errors for Company 9

growth model	mean estimate error
Pearl	5.3764
Linearized Fisher-Pry	60.3477
Gompertz	2.4724
Linearized Gompertz	6.6916

factors. The parameters estimated from each discount factor are
provided in Table 34, with five periods ahead forecast (from 1981 to
1985 year).

The mean forecast errors calculations indicate that the best model
uses a discount factor of 0.95. The resultant model is

$$Y(t) = e^{-(5.8782) \cdot e^{-(0.1196)t}} + \epsilon(t).$$

Figure 17 illustrates both the actual data and the best forecasted
data which employed a discount factor of 0.95.

Results of other companies

The nine data series are employed to test the algorithm for
selecting the appropriate model. The results of the two pre-fit
diagnostics and the discount factor for smallest forecasting error are
provided in Table 35.

The mean estimate errors for each of the four growth models as
applied to the nine time series data are stated in Appendix N. This
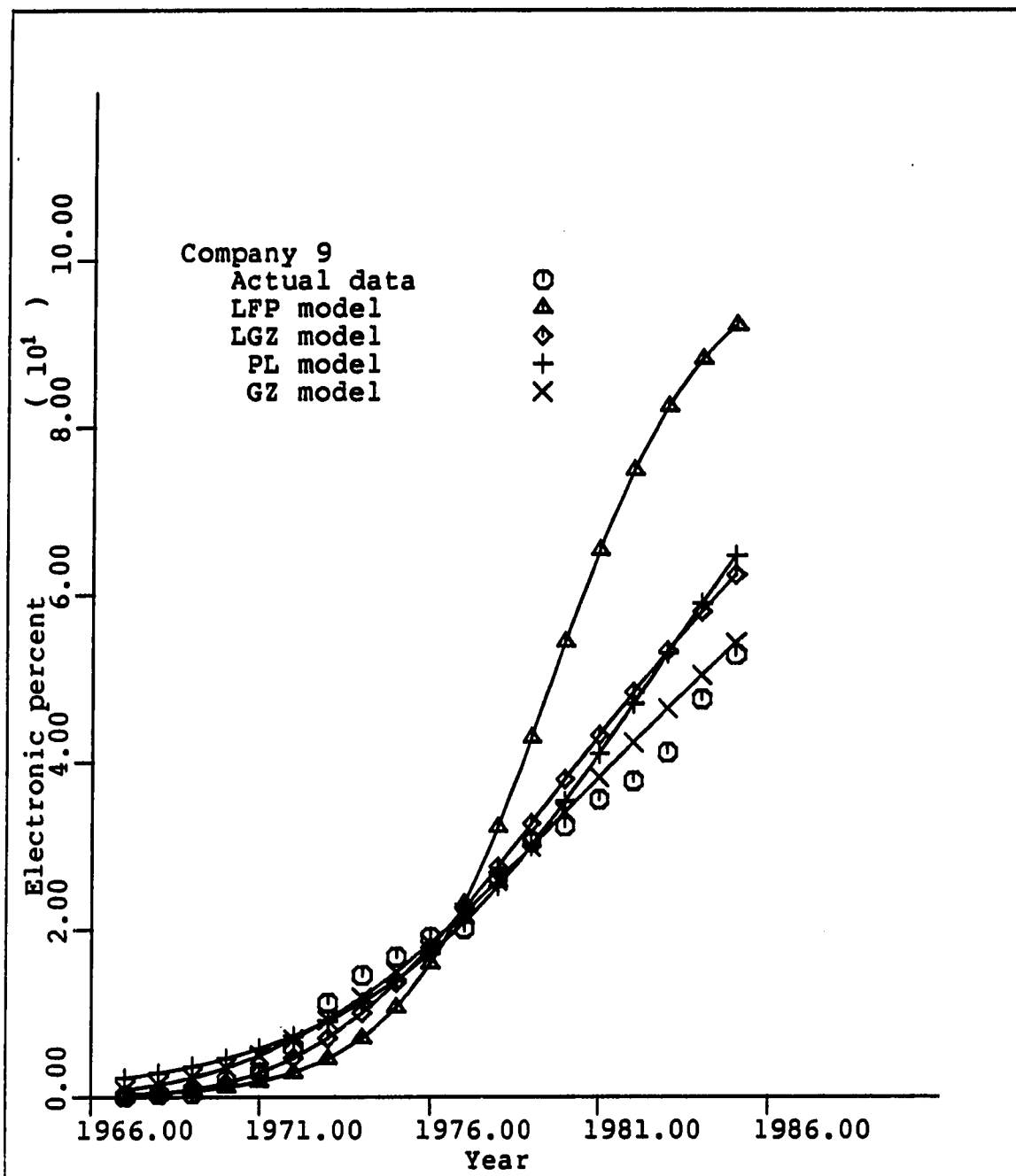
FIGURE 16. Actual vs forecasted with four growth models

TABLE 34. Best forecasting model for Company 9

| year | actual | discount factor | | | | |
		1.00	0.95	0.90	0.80	0.70
1981	35.51	38.18	37.62	37.24	37.05	37.66
1982	37.71	42.33	42.00	41.90	42.43	44.04
1983	41.12	46.41	46.32	46.48	47.69	50.23
1984	47.49	50.39	50.52	50.93	52.77	56.09
1985	52.80	54.23	54.56	55.20	57.58	61.54
mean forecast error		13.37	12.45*	13.40	23.72	55.62
	parameter G	5.2705	5.8782	6.6442	8.9729	13.4286
	parameter k	0.1133	0.1196	0.1271	0.1468	0.1747

* indicates the smallest mean forecast error

post-fit method (the lowest mean estimate errors is appropriate only in choosing the general form of the model) confirms the choice made by the plots, except Company 8. The actual data and the forecasted by four growth models are plotted in Appendix O. The parameter estimates from each discount factor are provided in Appendix P with forecasts of five periods ahead and the mean forecast errors for each level. Appendix Q provides the plot of the actual data against the forecasted data with a discount factor.

Note that no one type of model was selected as consistently best. This finding implies that consistently employing one type of growth model for any technological pattern would provide poorer results than following the systematic selection procedures. The range of results for choosing the discount factor, indicates that the need to determine
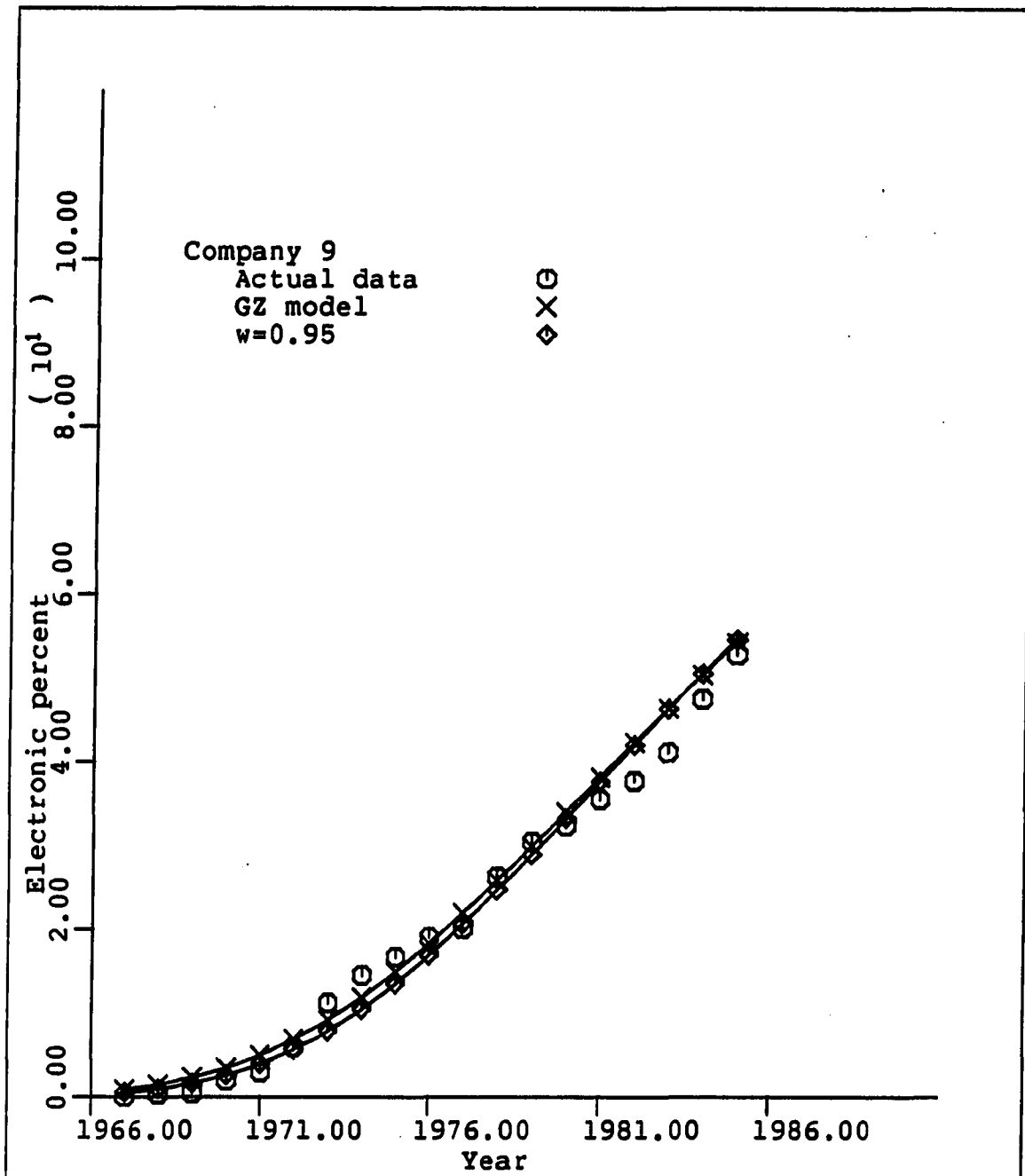
FIGURE 17. Actual vs forecasted with a discount factor

TABLE 35. Results of pre-fit diagnostics and discount factor

Company	general form	pseudo-residuals	best forecast model	discount factor
1	Logistic	constant	Pearl	0.95
2	Gompertz	constant	Gompertz	0.80
3	Logistic	increasing	Pearl	1.00
4	Gompertz	increasing	Gompertz	0.70
5	Logistic	constant	Pearl	1.00
6	Gompertz	increasing	Gompertz	1.00
7	Logistic	increasing	Pearl	0.90
8	Gompertz	increasing	Gompertz	1.00
10	Gompertz	increasing	Gompertz	0.80

the best factor in forecasting the curve is also a necessary step.
Consistent usage of only one discount factor, be it large or small,
would not provide the best forecasts in the long run.

Remaining Life Obtained From Growth Models

Depreciation accounting has as its main objective, the recovery of
the original cost of plant investment less net salvage, over the
estimated useful life of that plant. Accuracy of the whole life
technique in meeting this objective depends entirely on the original
estimates of service life and net salvage for an account. Where the
whole life technique has been used and original estimates prove
inaccurate, excessive or deficient accumulations in the depreciation
reserve frequently occur. To overcome this, the use of the remaining
life technique has been adopted by many utilities.

The remaining life technique compensates for prior life and salvage estimate inaccuracies by considering the effects of prior rates upon the depreciation reserve. This technique utilizes the remaining life in calculating the accrual rate for a calendar year. The accrual rate for a calendar year is applied to the uncovered cost less net salvage of the property at the beginning of the calendar year or it may be computed in such a way as to be applicable to the gross plant at the beginning of the year.

The main purpose of doing technological forecasting in life analysis is to develop life indicators for specific types of equipment. With the growth model selected, the remaining life can be estimated. For example, Table 36 gives the substitution of electronic switching as forecast by the growth model at one of the telephone companies used in this study.

Assuming the half-year convention, the remaining life at the beginning of 1984 can be calculated as:

$$\frac{\frac{1}{2}(100.0-84.9)+(100.0-86.5)+...+(100.0-99.7)+(100.0-100.0)}{(100.0-84.9)}$$

= 3.89 years.

The basic equation for the straight-line remaining life accrual with the vintage group is

$$D_x = [(B_{x-1})(1-S_f) - DR_{x-1}](\frac{1}{RL_{x-1}})$$

$$+ \frac{1}{2} C_x \cdot (1-S_a)(\frac{1}{PASL}) \tag{69}$$

TABLE 36. Percent of electronic switching at Company 3

year	% electronic	year	% electronic
1969	2.0	1984	84.9
1970	3.6	1985	86.5
1971	5.5	1986	89.8
1972	9.7	1987	92.4
1973	13.3	1988	94.4
1974	16.2	1989	95.8
1975	20.7	1990	97.0
1976	24.2	1991	97.8
1977	31.5	1992	98.4
1978	41.0	1993	98.8
1979	47.7	1994	99.2
1980	52.4	1995	99.4
1981	59.0	1996	99.6
1982	67.3	1997	99.7
1983	79.7	1998	100.0

where

D_x = depreciation accrual for calendar year x

B_{x-1} = vintage balance at beginning of calendar year x

DR_{x-1} = depreciation reserve balance at beginning of calendar year x

RL_{x-1} = remaining life of the vintage at the beginning of calendar year x

C_x = amount installed when x is the year of installation

PASL = probable average service life of the vintage

S_f = future net salvage ratio for the vintage

S_a = average net salvage ratio for the vintage.

The first term of the equation (69) is equal to zero for the year when the property in the vintage is placed in service; in subsequent years, this term computes the depreciation charge for the vintage. The second term computes the depreciation charge for the year when the property in the vintage is placed in service; in subsequent years, this term is vanished.

In addition to the proper adjustment for 100% recovery of capital investment, some other advantages of the remaining life technique over the whole life technique are

1. more closely matches capital recovery with consumption recognizing that there will normally be changes in estimated lives as retirement plans are solidified and future conditions are known,

2. reduce the risk from the results of obsolescence since proper capital recovery will more adequately fund modernization and replacement programs,

3. reduce the impact of inflation by improving the allocation of recovery over the remaining life of the investment,

4. reduce revenue requirements in the long run by providing for 100% capital recovery and reducing the rate base.

CHAPTER X.   CONCLUSIONS AND RECOMMENDATIONS

One of the telephone industry's greatest challenges for the eighties and beyond might be life estimates for increasingly obsolescent plant investments. Recent competitive and technological changes during the past decade have accelerated the need for better capital recovery methods. Competition and technology have together shortened the expected lives of property which could not have been forecasted several years ago.

Since the usage of technological growth models has been prevalent in various technological forecasting environments, the various forms of growth models have become numerous. Of six such models studied, some models do significantly better than others, especially at low penetration levels in predicting future levels of growth, although that performance cannot easily be linked to fitting ability. The lack of a direct relationship between fitting and forecasting ability implies that fitting alone should not be used a priori to select growth models for the purpose of forecasting.

A set of criteria for choosing an appropriate model for technological forecasting was developed. Two major characteristics of an S-shaped curve were elected which differentiate the various models; they are the skew of the curve and the underlying assumptions regarding the variance of the error structure of the model.

If the analyst wishes to fit properly the sample data with the model that represents the underlying population, then following the procedures would be appropriate.

1. Plot the linear transformation of data, assuming both the logistic model or the version of Gompertz curves, and choose the straighter of the two plots. The straighter line determines the general form of the model.

or

Find a mean estimate error. If the smallest mean estimate error is for the Pearl or the linearized Fisher-Pry model, the logistic version is chosen. If the Gompertz or the linearized Gompertz curve yields the smallest mean estimate error, then the Gompertz version is chosen.

2. Once the general form of the model has been chosen, plot the pseudo-residuals of the observed data from their three point moving averages. If the plots display a constant variance over time, choose the original form of model (i.e., the Pearl curve or the Gompertz curve). If the plots display an increasing variance over time, choose the transformed model (i.e., the linearized Fisher-Pry model or the linearized Gompertz curve).

However, if the analyst is more concerned with the forecasting ability, rather than the accuracy of the fitting ability, then following the procedures would be appropriate.

1. Plot the linear transformation of data, assuming both the logistic model or the version of Gompertz curves, and choose

the straighter of the two plots. The straighter line
determines the general form of the model.

or

Find a mean estimate error. If the smallest mean estimate
error is for the Pearl or the linearized Fisher-Pry model,
the logistic version is chosen. If the Gompertz or the
linearized Gompertz curve yields the smallest mean estimate
error, then the Gompertz version is chosen.

2. If the logistic form of the model has been chosen, the Pearl
   growth curve is utilized to forecast the data. If the
   Gompertz version of the model has been chosen, the Gompertz
   growth is utilized to forecast the data.

3. Once the population model has been chosen, use the discount
   least squares to derive the best estimates of the parameters
   of that model.

Numerous studies show that a lack of consensus exists among
analysts as well as a lack of consistency by the same analyst over time
relative to typical judgments made in the technological forecasting.
Therefore, successful application of objective decision-making tools in
any stage of the forecasting would appear to help lessen to some degree
these inconsistencies. Although the use of statistical techniques
still require some subjective input and interpretations, the analytical
procedure provides a more objective basis than would otherwise result.

Remaining life technique is suggested to better match the challenges of accelerated technology and competition within the regulated environment. The flexibility of the remaining life method will allow an even better chance to provide a complete recovery of the original cost, not more and not less, over the life of the property.

Although this study includes the theoretical statistical effects of continued use of the growth models with assumption violations, future research should be undertaken to isolate the effects of the violations included in the data sets occurring one at a time and in combination. Differing degrees and combinations of the violations should be simulated into the data set, and the various results observed and analyzed. Future research is necessary to resolve the range of the discount factor which would be valid as it ties into the forecasting of the growth curve and the different problems of incorporating technological forecasting fully into the assumptions of future additions and retirements patterns.

BIBLIOGRAPHY

1. Gold, H. The $26 billion solution. _Forbes_ 1985, 136, 40-41.

2. Robinson, T. D. An Economist's Perspective on Depreciation. _Telephony_ August 11, 1986, 211, 62-68.

3. Marston, A., Winfrey, R., and Hempstead, J. C. "Engineering Valuation and Depreciation." Ames, Iowa: The Iowa State University Press, 1982.

4. Fitch, W. C. and Wolf, F. Conceptual Framework for Forecasting the Useful Life of Industrial Property. Proceedings of the Iowa State University Regulatory Conference, Iowa State University, Ames, Iowa, 1984.

5. Wolf, F. Forecasting Forces of Mortality. Proceedings of the Iowa State University Regulatory Conference, Iowa State University, Ames, Iowa, 1985.

6. Ocker, A. C. Life Cycle Approach to Depreciation Analysis. Proceedings of the Iowa State University Regulatory Conference, Iowa State University, Ames, Iowa, 1983.

7. White, B. E. Economic Forces of Retirement. Proceedings of the Iowa State University Regulatory Conference, Iowa State University, Ames, Iowa, 1986.

8. Dandeker, M. Investigation the Product Life Cycle Concept: An Application to Capital Recovery, Evaluation within the Telephone Industry. Unpublished Ph. D. Dissertation. Iowa State University of Science and Technology, Ames, Iowa, 1987.

9. Kateregga, K. A. Technological Forecasting Models and Their Applications in Capital Recovery. Unpublished Ph. D. Dissertation. Iowa State Iowa State University of Science and Technology, Ames, Iowa, 1987.

10. Fitch, W. C., Wolf, F., and Bissinger, B. H. "The Estimation of Depreciation." Kalamazoo, Michigan: Depreciation Programs, Inc., 1982.

11. Lenz, R. C., Jr. Technological Forecasting. Report ASD-TDR-62-414. Aeronautical Systems Divisions, Wright-Patterson Air Force Base, Ohio, 1962.

12. Bright, J. R. "Practical Technological Forecasting: Concepts and Exercises." Austin, Texas: The Industrial Management Center, Inc., 1983.

13. Jantsch, E. Technological Forecasting in Corporate Planning. In "Technological Forecasting and Corporate Strategy." Edited by G. Wills et al. London: Bradford University Press and Crosby Lockwood and Sons Ltd., 1969.

14. Landford, H. W. Approaches to Technological Forecasting as a Planning Tool. Long Range Planning August, 1974, 7, 49-58.

15. Landford, H. W. "Technological Forecasting Methodologies." New York: American Management Association, Inc., 1972.

16. Markridakis, S., Wheelwright, S. C., and McGee, V. E. "Forecasting: Methods and Applications." 2nd ed. New York: John Wiley & Sons, 1983.

17. Kovac, F. J. Technological Forecasting-Tires. Chemical Technology 1971, 1, 18-23.

18. Grade, V. D. and Patel, R. R. Technological Forecasting for Power Generation: A Study Using the Delphi Technique. Long Range Planning 1985, 18, No. 4, 73-79.

19. Cetron, M. V. and Monohan, T. I. An Evaluation and Appraisal of Various Approaches to Technological Forecasting. In "Technological Forecasting for Industry and Government: Methods and Applications." Edited by J. R. Bright. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968.

20. Ayres, R. U. "Technological Forecasting and Long-Range Planning." New York: McGraw-Hill, 1969.

21. Chambers, J. C. "How to choose the Right Forecasting Technique." Harvard Business Review on Management. New York: Harper and Row, 1975.

22. Helmer, O. Prospects of Technological Progress. In "An Introduction to Technological Forecasting." Edited by J. P. Martino. New York: Gordon and Breach Science Publishers, 1972.

23. Helmer, O. and Rescher, N. On the Epistemology of the Inexact Science. Management Science 1959, 6, No. 1, 25-52.

24. Gordon, T. J. and Hayward, H. Initial Experiment with the Cross-Impact Matrix Method of Forecasting. Futures 1968, 1, No. 2, 100-116.

25. Helmer, O. Problems in Futures Research: Delphi and Causal Cross-Impact Analysis. _Futures_ 1977, 9, No. 1, 17-31.

26. Rochberg, R., Gordon, T. J., and Helmer, O. The Use of Cross-Impact Matrices for Forecasting and Planning. IFF Report. Middletown, Conn.: The Institute for the Future, April, 1970.

27. Linstone, H. A. and Sahal, D. "Technological Substitution." New York: Elsevier, 1976.

28. Ascher, W. "Forecasting an Appraisal for Policy Models and Planner." Baltimore, Md.: The Johns Hopkins University Press, 1978.

29. Bright, J. R. "A Brief Introduction to Technological Forecasting." Austin: Permaquid Press, 1972.

30. Kahn, H. "The Next 200 Years." New York: Morrow, 1976.

31. Zwicky, F. and Wilson, G. "New Methods of Thought and Procedure." New York: Springer Verlag, 1967.

32. Forrester, J. W. Industrial Dynamics: A Major Breakthrough for Decision Makers. _Harvard Business Review_ 1958, 36, 37-66.

33. Lenz, R. C., Jr. and Landford, H. W. The Substitution Phenomenon. _Business Horizons_ 1971, 15, No. 1, 63-68.

34. Martino, J. P. "Technological Forecasting for Decision Making." New York: Elsevier, 1975.

35. Lakhani, H. Diffusion of Environment-Saving Technological Change: A Petroleum Refining Case Study. _Technological Forecasting and Social Change_ 1975, 7, No. 1, 33-55.

36. Mansfield, E. "Industrial Research and Technological Innovation: An Econometric Analysis." New York: W. W. Norton & Company, Inc., 1968.

37. Pearl, R. "The Biology of Population." New York: Alfred A. Knopf, 1925.

38. Pearl, R. and Reed, L. J. A Further Note on the Mathematical Theory of Population Growth. _Proceedings of the National Academy of Sciences_ 1922, 8, 365-368.

39. Fisher, J. C. and Pry, R. H. A Simple Substitution Model of Technological Change. _Technological Forecasting and Social Change_ 1971, 3, 75-88.

40. Mansfield, E. Technical Change and the Rate of Imitation. *Econometrica* 1961, 29, 741-766.

41. Blackman, A. W., Jr. The Market Dynamics of Technological Substitutions. *Technological Forecasting and Social Change* 1974, 6, 41-63.

42. Blackman, A. W., Jr. The Rate of Innovation in the Commercial Jet Engine Market. *Technological Forecasting and Social Change* 1971, 2, 268-296.

43. Blackman, A. W., Jr. A Mathematical for Trend Forecasts. *Technological Forecasting and Social Change* 1972, 3, 441-452.

44. Bass, F. M. A New Product Growth Model for Consumer Durables. *Management Science* 1969, 15, No. 5, 215-227.

45. Haines, G. H., Jr. A Theory of Market Behavior after Innovation. *Management Science* 1964, 10, No. 4, 634-658.

46. Faurt, L. A. and Woodlock, J. W. Early Prediction of Market Success for New Grocery Products. *Journal of Marketing* 1960, 25, 31-38.

47. Nevers, J. V. Extensions of a New Product Growth Model. *Sloan Management Review* 1972, 13, No. 2, 72-91.

48. Heeler, R. M. and Hustad, T. P. Problems in Predicting New Product Growth for Consumer Durables. *Management Science* 1980, 26, 1007-1020.

49. Eide, K. and Ness, R. "Estimation Algorithm." Krannert Graduate School, Purdue University, 1974.

50. Booth, H. Transforming Gompertz's Function for Fertility Analysis: The Development of a Standard for the Relational Gompertz Function. *Population Studies* 1984, 38, 495-506.

51. Floyd, A. L. A Methodology for Trend-Forecasting of Figures of Merit. In "Technological Forecasting for Industry and Government: Methods and Applications." Edited by J. R. Bright. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968.

52. Sharif, M. N. and Uddin, G. A. A Procedure for Adapting Technological Forecasting Models. *Technological Forecasting and Social Change* 1975, 7, 99-106.

53. Sharif, M. N. and Kabir, C. A Generalized Model for Forecasting Technological Substitution. *Technological Forecasting and Social Change* 1976, 8, 353-364.

54. Kao, J. H. K. A Graphical Estimation on Mixed Weibull Parameters in Life-Testing of Electron Tubes. Technometrics 1959, 1, 389-407.

55. Lieblein, J. and Zelen, M. Statistical Investigation of the Fatigue Life of Deep-Groove Ball Bearings. Research of the National Bureau of Standards Journal 1956, 57, 273-316.

56. Weibull, W. A Statistical Distribution Function of Wide Applicability. Journal of Applied Mechanics 1951, 18, 293-297.

57. Peto, R., Lee, P. N., and Page, W. S. Statistical Analysis of the Bioassay of Continuous Carcinogens. The British Journal of Cancer 1972, 26, 258-261.

58. Sharif, M. N. and Islam, M. N. The Weibull Distribution as a General Model for Forecasting Technological Change. Technological Forecasting and Social Change 1980, 18, 247-256.

59. Beck, P. J. A Critical Analysis of the Regression Estimation in Audit Sampling. Journal of Accounting Research 1980, 16-37.

60. Baillie, R. T. The Asymptotic Mean Squared Error of Multistep Prediction from the Regression Model with Autoregressive Errors. Journal of the American Statistical Association 1979, 74, No. 365, 175-184.

61. Fox, J. "Linear Statistical Models and Related Methods." New York: John Wiley & Sons Inc., 1984.

62. Draper, N. R. and Smith, H. "Applied Regression Analysis." New York: John Wiley & Sons Inc., 1981.

63. Davies, O. L. and Oliver, F. R. "Design and Analysis of Industrial Experiments." New York: Hafner Pub. Co., 1956.

64. Hartley, H. O. The Modified Gauss-Newton Method for Fitting Non-linear Regression Function by Least Squares. Technometrics 1961, 3, No. 2, 269-280.

65. Marquardt, D. W. An Algorithm for Least Squares Estimation of Non-linear Parameter. Journal of Society for Industrial and Applied Mathematics 1963, 11, 431-441.

66. Cochran, D. and Orcutt, G. H. Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms. Journal of the American Statistical Association 1949, 44, No. 245, 32-61.

67. Gilchrist, W. G. "Statistical Forecasting." Chichester, England: John & Sons, 1976.

68. Brown, R. G. "Smoothing, Forecasting, and Prediction of Time Series." Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962.

69. Hayes, J. G. "Numerical Approximation to Functions and Data." University of London: The Athlone Press, 1970.

70. Shapiro, S. S. and Wilk, M. B. An Analysis of Variance Test for Normality: Complete Samples. Biometrika 1965, 52, 591-611.

71. Shapiro, S. S., Wilk, M. B., and Chen, H. J. A Comparative Study of Various Tests for Normality. Journal of the American Statistical Association 1968, 63, No. 324, 1343-1372.

72. Goldfeld, S. M. and Quandt, R. E. Some Tests for Homoscedasticity. Journal of the American Statistical Association 1965, 60, No. 310, 539-547.

73. Durbin, J. and Watson, G. S. Testing for Serial Correlation in Least Squares Regression. Biometrika 1951, 38, 159-178.

74. Berenson, M. L., Levine, D. M., and Goldstein, M. "Intermediate Statistical Methods and Applications." Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1983.

75. Nagar, A. L. Statistical Testing of the Accuracy of Forecasts. Statistica Neerlandica 1962, 16, No. 3, 237-249.

76. Eilon, S., Roger, T., and Gold, B. Measuring the Quality of Economic Forecasts. Omega 1973, 1, No. 2, 217-227.

77. Snedecor, G. W. and Cochran, W. G. "Statistical Methods." Seventh ed. Ames, Iowa: The Iowa State University Press, 1980.

78. May, J. M. Extended and Corrected Tables of the Upper Percentage Points of the Studentized Range. Biometrika 1952, 39, 192-193.

79. Conover, W. J. "Practical Nonparametric Statistics." 2nd ed. New York: John Wiley & Sons, 1980.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to Dr. Harold A. Cowles for his steady guidance and encouragement throughout the whole doctoral process. A debt of gratitude is also due the other members of my graduate committee, professors Thomas A. Barta, Charles P. Cox, William Q. Meeker, and Geraldine M. Montag. ·

I wish to express both my love and thanks to my wife, Ha-Youn, and my two sons, Jong-Rok and Kyung-Rok, who with love and understanding helped me to endure the tensions and hardships experienced during the course of my graduate work.

Finally, a special gratitude is due my mother, Hee-Kyung, and my father, Hyung-Ok Oh, who are longing for my family in Korea. Without their patience, support, and encouragement, this study would have been terminated. The least I can do is to dedicate the dissertation to my parents.

APPENDIX A.   AN ALTERNATE FORM OF THE FISHER-PRY MODEL

To derive the alternate form of the Fisher-Pry model, begin with the form of the equation (3).

$$Y = \frac{1}{2}[1 + \tanh a(t - t_0)]$$

$$= \frac{1}{2}[1 + \frac{\sinh a(t - t_0)}{\cosh a(t - t_0)}]$$

$$= \frac{1}{2}\{1 + \frac{0.5[e^{a(t - t_0)} - e^{-a(t - t_0)}]}{0.5[e^{a(t - t_0)} + e^{-a(t - t_0)}]}\}$$

$$= \frac{e^{a(t - t_0)}}{e^{a(t - t_0)} + e^{-a(t - t_0)}}$$

then

$$1 - Y = 1 - \frac{e^{a(t - t_0)}}{e^{a(t - t_0)} + e^{-a(t - t_0)}}$$

$$= \frac{e^{-a(t - t_0)}}{e^{a(t - t_0)} + e^{-a(t - t_0)}}$$

It follows that,

$$\frac{Y}{1 - Y} = \frac{e^{a(t - t_0)}}{e^{a(t - t_0)} + e^{-a(t - t_0)}} \times \frac{e^{a(t - t_0)} + e^{-a(t - t_0)}}{e^{-a(t - t_0)}}$$

$$= \frac{e^{a(t - t_0)}}{e^{-a(t - t_0)}}$$

$$= e^{2a(t - t_0)}$$

APPENDIX B. DERIVATION OF THE FISHER-PRY MODEL

Given the form of the Pearl growth curve,

$$Y = \frac{L}{1 + ae^{-\beta t}}$$

if $L = 1.0$, then

$$Y = \frac{1}{1 + ae^{-\beta t}}$$

And

$$1 - Y = \frac{ae^{-\beta t}}{1 + ae^{-\beta t}}$$

It follows that

$$\frac{Y}{1 - Y} = \frac{1}{1 + ae^{-\beta t}} \times \frac{1 + ae^{-\beta t}}{ae^{-\beta t}}$$

$$= \frac{e^{\beta t}}{a}$$

When $t = t_0$, in which the new technology captures 50 percents of the usage, i.e., $Y = 0.5$,

$$\frac{Y}{1 - Y} = \frac{0.5}{1 - 0.5}$$

$$= 1.0$$

$$= \frac{e^{\beta t_0}}{a}$$

so,

$$a = e^{\beta t_0}$$

By substitution of a,

$$\frac{Y}{1-Y} = \frac{e^{\beta t}}{e^{\beta t_0}}$$

$$= e^{\beta(t - t_0)}$$

If let $\beta = 2a$, where a is the half the annual fractional growth in the early years, then,

$$\frac{Y}{1-Y} = e^{2a(t - t_0)}$$

APPENDIX C.   DERIVATION OF THE BASS MODEL

Given the form of the Pearl growth curve,

$$Y = \frac{L}{1 + ae^{-\beta t}}$$

the Bass model may be obtained by considering the derivative of Y with respect to time.   It follows that,

$$S(t) = \frac{dY}{dt}$$

$$= \frac{La\beta e^{-\beta t}}{[1 + ae^{-\beta t}]^2}$$

$$= \frac{\beta}{L} \left[\frac{L}{1 + ae^{-\beta t}}\right] \times \left[L - \frac{L}{1 + ae^{-\beta t}}\right]$$

$$= \frac{\beta}{L} Y [L - Y]$$

$$= \beta Y - \frac{\beta}{L} Y^2$$

$$\equiv c_0 + c_1 Y + c_2 Y^2$$

where $c_0 = 0$, $c_1 = \beta$ of the Pearl growth curve, and $c_2 = (-1/L)\beta$ of the Pearl growth curve for the error free Bass model.

APPENDIX D. LIST PROGRAM FOR CALCULATING D-W STATISTIC

```
//CURVE1  JOB  I3583,HYUN,MSGLEVEL=1
/*JOBPARM LINES=20,BIN=407,DUPLEX=NO,FLASH=NONE
//S1 EXEC SAS,TIME=10
//INDAT  DD  DSN=H.I3583.CURVE1,UNIT=DISK,DISP=SHR
//SYSIN  DD  *
*                        ;
*     READ DATA SET      ;
*                        ;
DATA CURVE1;
  INFILE INDAT;
  INPUT X Y;
*                                  ;
*     FITTING WITH PEARL CURVE     ;
*                                  ;
DATA PEARL;
  SET CURVE1;
  IF Y>=1.0 THEN Y=0.999999;
  IF Y<=0.0 THEN DELETE;
PROC NLIN DATA=PEARL ITER=100 BEST=10 METHOD=MARQUARDT;
  PARMS PALPHA=40.774 PBETA=0.1811;
  P1=EXP(-PBETA*X);
  P2=1.0/(1.0+PALPHA*P1);
MODEL Y=P2;
  DER.PALPHA=-P1*P2*P2;
  DER.PBETA=PALPHA*X*P1*P2*P2;
  OUTPUT OUT=PEARLO P=PEARLP R=PEARLR;
*                                            ;
*     CALCULATE DURBIN-WATSON STATISTIC      ;
*                                            ;
DATA SETP;
  SET PEARLO;
  KEEP PEARLR;
PROC MATRIX;
  FETCH ERROR DATA=SETP;
  SSE=ERROR'*ERROR;
  TEM=ERROR(1:22,)-ERROR(2:23,);
  NUM=TEM'*TEM;
  DWD=NUM#/SSE;
  R=ERROR(1:22,)'*ERROR(2:23,)#/SSE;
PRINT DWD R;
*                                      ;
*     FITTING WITH GOMPERTZ CURVE      ;
*                                      ;
DATA GOMP;
  SET CURVE1;
  IF Y>=1.0 THEN Y=0.999999;
  IF Y<=0.0 THEN DELETE;
```

```
PROC NLIN DATA=GOMP ITER=100 BEST=10 METHOD=MARQUARDT;
  PARMS GOMPG=7.532 GOMPK=0.1254;
    G1=-GOMPK*X;
    G2=EXP(G1);
    G3=-GOMPG*G2;
    G4=EXP(G3);
MODEL Y=G4;
  DER.GOMPG=-G2*G4;
  DER.GOMPK=GOMPG*X*G2*G4;
  OUTPUT OUT=GOMPO P=GOMPP R=GOMPR;
*                                              ;
*       CALCULATE DURBIN-WATSON STATISTIC      ;
*                                              ;
DATA SETG;
  SET GOMPO;
  KEEP GOMPR;
PROC MATRIX;
  FETCH ERROR DATA=SETG;
  SSE=ERROR'*ERROR;
  TEM=ERROR(1:22,)-ERROR(2:23,);
  NUM=TEM'*TEM;
  DWD=NUM#/SSE;
  R=ERROR(1:22,)'*ERROR(2:23,)#/SSE;
PRINT DWD R;
*                                              ;
*       FITTING WITH WEIBULL CURVE             ;
*                                              ;
DATA WB;
  SET CURVE1;
  IF Y>=1.0 THEN Y=0.999999;
  IF Y<=0.0 THEN DELETE;
PROC NLIN DATA=WB ITER=100 BEST=10 METHOD=MARQUARDT;
  PARMS WALPHA=31.951 WBETA=2.260;
    W1=X/WALPHA;
    W2=W1**WBETA;
    W3=EXP(-W2);
    W4=1.0/WALPHA;
MODEL Y=1.0-W3;
  DER.WALPHA=-WBETA*W2*W3*W4;
  DER.WBETA=W2*W3*LOG(W1);
  OUTPUT OUT=WBO P=WBP R=WBR;
*                                              ;
*       CALCULATE DURBIN-WATSON STATISTIC      ;
*                                              ;
DATA SETW;
  SET WBO;
  KEEP WBR;
PROC MATRIX;
  FETCH ERROR DATA=SETW;
  SSE=ERROR'*ERROR;
```

```
    TEM=ERROR(1:22,)-ERROR(2:23,);
    NUM=TEM'*TEM;
    DWD=NUM#/SSE;
    R=ERROR(1:22,)'*ERROR(2:23,)#/SSE;
PRINT DWD R;
```

APPENDIX E. LIST OF DATA AND THEIR SOURCES

The abbreviation HSUS refers to: "Historical Statistics of the United States: Colonial Times to 1970." U.S. Bureau of the Census, Washington, D.C., 1975. The abbreviation SAUS refers to: "Statistical Abstract of the United States: U.S. Bureau of the Census, Washington, D.C., various years.

1. Rayon and nylon for cotton as tire cord in tire manufacture (1938-1962).

   Source: F. J. Kovac. "Tire Technology." 5th Ed. The Goodyear Tire & Rubber Co., Akron, Ohio, 1978, pp. 153-155.

2. Nylon, polyester and fiberglass for rayon and cotton as tire cord in tire manufacture (1962-1972).

   Source: F. J. Kovac. "Tire Technology." 5th Ed. The Goodyear Tire & Rubber Co., Akron, Ohio, 1978, pp. 153-155.

3. Catalytic and hydro-cracking for thermal cracking in crude oil processing (1938-1966).

   Source: see bibliography reference under H. Lakhani.

4. Steam and motor for sail in the United Kingdom registered shipping (1918-1938).

   Source: B. R. Mitchell. "Abstract of British Historical Statistics." Cambridge University Press, Cambridge, U.K., 1962, pp. 217-219.

5. Percent of underground bituminous coal automatically loaded (1923-1970).

Source: "Bituminous Coal Facts." National Coal Assoc., Washington, D.C., 1972, pp. 53.

6. Diesel for coal and fuel oil consumption on American railroads (1938-1970).

   Source: HSUS, Part II, pp. 783-739.

7. Percent of independent telephone companies connecting with the Bell system (1899-1957).

   Source: HSUS, Part II, pp. 783.

8. Open hearth for bessemer in raw steel production in the United States (1876-1960).

   Source: "Agricultural Statistics." U.S. dept. of Agriculture, Washington D.C., various years.

9. Percentage of U.S. corn acreage planted with corn hybrids (1933-1960).

   Source: "Agricultural Statistics." U.S. Dept. of Agriculture, Washington, D.C., various years.

10. Diesel for steam locomotives (1939-1962).

    Source: "Transport Statistics in the U.S." and "Statistics of Railroads in the U.S." Interstate Commerce Commission, Washington, D.C., various years.

11. Percentage of Pennsylvania anthracite mined by stripping (1927-1976).

    Source: "Minerals Yearbook - Mineral Fuels." 1965, Vol. II, for data 1927-1965. For data 1966-1976, "Minerals

Yearbook." U.S. Bureau of Mines, Washington, D.C., various years.

12. Steam and motor for sail in the U.S. Merchant Marine (1820-1960).

    Source: HSUS, Part II, pp. 748-750.

13. Basic oxygen process for bessemer and open hearth in raw steel production in the U.S. (1955-1981).
    Source: W. T. Lankford, Jr. "The Making, Shaping and Treatment of Steel." 10th ed. U.S. Steel, Pittsburgh, PA., 1985, pp.1508.

14. Color for B&W television in the United Kingdom (1968-1984).
    Source: "Annual Abstract of Statistics." Central Statistics Office, HM's Stationery Office, London, various years.

15. Percentage of iron ore pelletized in the U.S. (1953-1973).
    Source: "Minerals Yearbook." U.S. Bureau of Mines, Washington, D.C., various years.

16. Percentage of farm dwelling units with electric service (1920-1956).
    Source: HSUS, Part II, pp. 827.

17. By-product coke for oven coke in the U.S. (1900-1962).
    Source: E. T. Sheridan and J. A. DeCarlo. "Coal Carbonization in the U.S.: 1900-1962." U.S. Bureau of Mines Information Circular 8251, 1965, pp. 60.

18. Percentage of households in the U.S. with a television set (1946-1980).

    Source: SAUS, various years.

19. Percentage of households in the U.S. with a color television set (1955-1984).

    Source: SAUS, various years.

20. Percentage of households in the U.S. with a radio receiver (1927-1970).

    Source: HSUS, Part II, pp. 796. For number of households, see HSUS, Part I, pp. 43.

21. Percentage of homes in the U.S. with at least a mechanical refrigerator (1925-1952).

    Source: J. F. Dewhurst and Associates. "America's Needs and Resources: A New Survey." The Twentieth Century Fund, New York, 1955, pp. 1041.

22. Basic oxygen for bessemer and open hearth pig iron total consumption in the U.S. (1957-1984).

    Source: Annual Statistical REport." American Iron and Steel Institute, New York, various years.

APPENDIX F. FITTING ERROR AT EACH PENETRATION LEVEL

TABLE 37. Mean estimate errors at 5% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	0.01223	0.00554	0.00143	0.00519	0.00323	0.00059
2	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.10225	0.02912	0.00273	0.01436	0.00940	0.00245
4	0.00933	0.00129	0.00095	0.00266	0.00102	0.00085
5	0.00665	0.01441	0.01614	0.00190	0.00510	0.00343
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
7	0.01533	0.02103	0.02274	0.00677	0.01074	0.00858
8	0.00012	0.00010	0.00000	0.00012	0.00010	0.00000
9	0.00103	0.00080	0.00020	0.00003	0.00024	0.00009
10	0.00330	0.00152	0.00055	0.00236	0.00130	0.00050
11	0.01346	0.01348	0.01374	0.01330	0.01340	0.01360
12	0.02823	0.02826	0.03025	0.02470	0.02620	0.02660
13	0.01881	0.00644	0.01116	0.00900	0.00570	0.00710
14	0.02155	0.00145	0.00037	0.00100	0.00010	0.00000
15	0.01334	0.01214	0.00950	0.01120	0.01090	0.00810
16	0.00108	0.00069	0.00065	0.00090	0.00060	0.00060
17	0.01474	0.01975	0.01931	0.01380	0.01530	0.01420
18	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
19	0.01814	0.00368	0.00001	0.00060	0.00020	0.00000
20	0.01990	0.00185	0.00003	0.00170	0.00030	0.00000
21	0.01191	0.01291	0.01325	0.00930	0.01220	0.01020
22	0.01513	0.01459	0.01565	0.01410	0.01430	0.01550

Key: LFP = linearized Fisher-Pry model
LGZ = linearized Gompertz growth curve
LWB = linearized Weibull growth curve
PL = Pearl growth curve
GZ = Gompertz growth curve
WB = Weibull growth curve

TABLE 38. Mean estimate errors at 10% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	0.03455	0.00969	0.00112	0.00866	0.00414	0.00050
2	0.22212	0.11435	0.05602	0.09964	0.07615	0.03203
3	0.13494	0.02825	0.07052	0.02176	0.02279	0.03142
4	0.01022	0.00435	0.00433	0.00244	0.00373	0.00326
5	0.05459	0.09189	0.09610	0.00855	0.02132	0.01170
6	0.02740	0.01666	0.01384	0.02266	0.01583	0.01327
7	0.06476	0.09483	0.10022	0.00699	0.01466	0.00872
8	0.13344	0.12145	0.07760	0.12262	0.11688	0.07674
9	0.00890	0.01500	0.00064	0.00015	0.00165	0.03400
10	0.01448	0.00415	0.00446	0.00720	0.00380	0.00420
11	0.06503	0.07042	0.07467	0.04930	0.06000	0.05430
12	0.11721	0.07691	0.07007	0.09160	0.07260	0.06710
13	0.04631	0.00575	0.00840	0.00840	0.00540	0.00530
14	0.05270	0.01784	0.02054	0.00120	0.00790	0.00510
15	0.12828	0.10963	0.12245	0.11890	0.10200	0.11020
16	0.00476	0.01237	0.01692	0.00360	0.00970	0.01020
17	0.47176	0.12028	0.13728	0.07210	0.05500	0.04790
18	0.00806	0.00441	0.00119	0.00050	0.00060	0.00000
19	0.01923	0.02165	0.02998	0.00400	0.00820	0.00640
20	0.01990	0.00185	0.00003	0.00170	0.00030	0.00000
21	0.00645	0.01051	0.01015	0.00600	0.00780	0.00640
22	0.02290	0.00976	0.01247	0.01190	0.00940	0.01140

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

TABLE 39. Mean estimate errors at 25% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	0.36894	0.12080	0.11748	0.17172	0.11714	0.11610
2	0.26045	0.38305	0.63486	0.25328	0.37137	0.44676
3	0.21047	0.09414	0.29173	0.04892	0.08601	0.09575
4	1.68377	0.55596	0.29564	0.42935	0.33376	0.23439
5	0.12630	0.45140	0.40429	0.02309	0.05751	0.02491
6	0.12841	0.06331	0.01986	0.08111	0.05247	0.01890
7	0.04964	0.16993	0.12332	0.03664	0.02132	0.02826
8	0.17978	0.24008	0.30124	0.17434	0.22901	0.24898
9	0.91505	0.01855	0.11421	0.07383	0.00857	0.03400
10	0.02982	0.05904	0.04876	0.02259	0.05120	0.03867
11	0.32704	0.24372	0.21875	0.24487	0.22795	0.21655
12	0.95822	0.54617	0.37144	0.52367	0.43203	0.29199
13	0.04022	0.11473	0.03468	0.01403	0.05725	0.02305
14	0.26551	0.06961	0.01111	0.00798	0.01041	0.00363
15	2.48735	1.00619	1.32694	1.17363	0.86023	0.89379
16	0.21170	0.19457	0.19522	0.18272	0.19455	0.19311
17	2.52929	0.30349	0.29992	0.18230	0.10540	0.08134
18	0.27331	0.04157	0.08930	0.01791	0.00438	0.00655
19	0.30120	0.67475	0.23324	0.20226	0.11591	0.20152
20	1.97013	0.09416	0.08827	0.11430	0.01579	0.00634
21	0.27502	0.02963	0.09631	0.11811	0.02930	0.09743
22	0.03080	0.13916	0.06527	0.02879	0.09400	0.04241

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

TABLE 40. Mean estimate errors at 50% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	3.69468	1.08279	1.67305	1.89527	1.06404	1.38847
2	4.07384	1.44985	1.41570	2.24166	1.33497	1.26091
3	0.21965	1.23263	1.29555	0.17174	0.48914	0.26004
4	1.10743	0.77732	0.96353	0.41338	0.74881	0.66593
5	0.39657	2.16547	1.38213	0.16472	0.26835	0.19733
6	1.08283	1.93794	1.60293	0.66565	1.40013	0.74674
7	1.84549	0.10868	0.94892	0.30886	0.05680	0.22121
8	0.32269	0.82901	0.86995	0.23358	0.62370	0.38010
9	4.82002	0.08191	0.77352	0.47547	0.05086	0.15662
10	0.14765	0.90519	0.62160	0.08252	0.45241	0.15011
11	0.63931	0.28109	0.30416	0.37720	0.27241	0.29184
12	1.03151	0.76488	0.85104	0.73974	0.76474	0.78736
13	0.36282	0.78819	0.20817	0.17669	0.14506	0.19204
14	4.41249	0.24572	0.94244	0.83646	0.22998	0.48857
15	6.04469	1.58830	2.17976	2.00264	1.09405	0.91039
16	0.28037	0.82316	0.32029	0.27636	0.57357	0.29230
17	3.02609	0.25273	0.25029	0.18395	0.21884	0.19716
18	4.94099	0.43755	3.52561	0.95216	0.27833	0.56649
19	2.59713	0.65814	0.90887	0.43791	0.15551	0.40688
20	5.70878	0.62487	1.17618	0.70447	0.21328	0.12258
21	0.68464	0.13025	0.11078	0.21217	0.12973	0.09743
22	0.08743	0.91858	0.49026	0.05371	0.29866	0.08530

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

TABLE 41. Mean estimate errors at 75% penetration level

| | | | models | | | |
case	LFP	LGZ	LWB	PL	GZ	WB
1	7.15601	2.36336	3.67231	3.96851	2.26093	2.83688
2	4.08234	1.25270	1.24735	2.06589	1.10956	1.01120
3	0.28249	2.77837	0.65994	0.27790	0.56840	0.40531
4	1.72684	0.67516	0.85764	0.68669	0.58931	0.56791
5	0.38282	2.52772	1.26106	0.28193	0.26485	0.38526
6	3.28521	1.90938	3.04221	2.69503	1.61057	2.73090
7	4.45628	0.47941	1.90795	1.11745	0.47926	0.70552
8	0.57547	2.31806	1.51720	0.36401	1.43925	0.50588
9	8.26574	0.56065	1.86954	1.63225	0.25676	0.50203
10	0.32227	2.71886	0.99563	0.11640	0.94761	0.15311
11	0.62607	0.90848	0.74769	0.50772	0.86988	0.66738
12	0.84037	1.51619	1.23293	0.72212	1.36977	0.96118
13	2.75080	0.87627	1.95286	1.32588	0.45545	1.32911
14	11.04635	1.47395	3.70980	2.91608	0.91838	1.32691
15	4.48695	1.23344	0.95608	1.60513	0.99680	0.87802
16	0.60328	2.39403	1.62302	0.49338	1.42866	0.57671
17	2.30497	0.94895	0.77551	0.42943	0.85980	0.61079
18	6.47256	0.55576	5.79164	1.51362	0.26211	0.84561
19	4.99639	0.52712	2.33924	0.62513	0.13761	0.56415
20	8.20880	1.44659	2.82317	2.67443	0.99575	1.02232
21	6.03430	2.26356	3.03666	3.91492	2.10141	2.51096
22	3.53813	1.24465	2.59840	2.30110	0.98811	2.10834

Key: LFP = linearized Fisher-Pry model
LGZ = linearized Gompertz growth curve
LWB = linearized Weibull growth curve
PL = Pearl growth curve
GZ = Gompertz growth curve
WB = Weibull growth curve

TABLE 42. Mean estimate errors at 100% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	5.21653	5.57503	2.43570	3.10474	2.60882	2.34244
2	4.71180	0.62040	1.76970	1.18862	0.59549	0.61357
3	4.07601	1.15986	1.15699	1.96414	1.12989	1.10047
4	0.27829	6.84939	9.85136	0.23481	0.69785	0.39368
5	4.50413	0.74751	1.06305	1.45584	0.70946	1.06306
6	1.21003	8.78509	5.61189	0.67972	0.59904	0.96473
7	4.28618	1.58202	1.90577	1.58247	0.78358	1.04956
8	3.00954	1.78947	4.76722	1.64421	1.45700	1.77105
9	16.67182	1.94497	6.03498	3.09445	0.81952	1.28062
10	3.58012	17.71217	7.25945	0.18854	1.53513	0.31300
11	1.29810	2.69920	7.13774	1.21573	2.30180	2.25530
12	3.16856	9.50664	10.65070	1.70572	4.07890	2.45530
13	8.63899	1.43380	3.94384	3.17020	1.08866	2.25936
14	15.84705	2.46771	6.04870	4.11003	1.34514	1.85003
15	4.92750	1.25029	10.20264	1.82310	1.03011	0.91661
16	2.59194	10.27577	16.16847	1.16904	3.37391	1.42204
17	2.16336	7.70913	10.14813	1.63352	3.36956	2.33802
18	10.71234	1.65592	6.76328	1.57721	0.42468	0.79218
19	12.12109	0.39238	6.58546	1.44754	0.32282	1.23112
20	11.58502	2.13464	4.39264	3.35684	1.24588	1.19470
21	6.59566	2.10090	2.16837	3.90203	1.98753	2.03572
22	9.91564	2.72964	3.74656	4.83822	2.14119	2.88503

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

APPENDIX G. FORECASTING ERROR AT EACH PENETRATION LEVEL

TABLE 43. Mean forecast errors at 5% penetration level

case	LFP	LGZ	models LWB	PL	GZ	WB
1	· 58.096	4.319	98.577	33.346	12.226	149.659
2	134.815	35.495	24.301	134.815	35.495	24.301
3	33.308	62.658	267.214	12.832	138.047	272.717
4	123.429	10.935	9.551	97.287	7.956	12.952
5	16.894	218.992	120.614	1.835	113.068	9.632
6	6.029	83.151	521.688	6.029	83.151	521.688
7	25.406	62.064	15.617	87.990	2.226	77.204
8	1.759	44.445	416.669	1.496	45.691	416.611
9	74.316	2.521	47.412	66.349	3.890	52.760
10	15.072	56.833	153.135	6.026	65.792	160.033
11	45.445	91.124	144.854	30.672	84.048	139.440
12	24.555	34.391	95.617	99.298	7.663	3.426
13	39.541	29.246	26.223	15.244	40.370	12.847
14	97.923	4.126	21.686	574.798	8.125	13.250
15	20.802	67.760	197.301	29.790	118.317	215.572
16	5.685	59.019	103.902	3.448	61.148	102.894
17	224.024	42.325	196.499	214.944	65.764	211.124
18	41.464	1.078	24.273	41.464	1.078	24.273
19	35.325	96.814	285.834	25.660	194.108	280.930
20	100.592	28.286	32.057	82.921	22.182	30.395
21	37.334	11.834	20.707	63.431	3.233	49.436
22	31.812	18.977	215.954	22.543	24.797	210.656

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

TABLE 44. Mean forecast errors at 10% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	34.680	13.231	116.033	14.335	28.193	141.054
2	54.244	1.428	56.684	17.704	8.661	124.994
3	6.711	86.748	273.713	8.003	109.126	164.091
4	120.268	14.094	51.880	104.278	17.986	64.473
5	7.202	170.596	48.790	9.254	25.631	13.250
6	5.246	14.423	19.578	3.116	17.682	25.638
7	7.954	23.630	22.468	122.330	19.675	137.321
8	9.313	87.041	383.982	31.182	116.079	391.378
9	72.154	4.108	62.753	64.054	12.034	66.245
10	1.156	93.301	132.535	6.253	98.326	123.326
11	164.156	34.011	119.812	212.517	66.393	200.302
12	113.771	12.897	9.858	74.979	9.316	5.993
13	26.304	35.565	10.582	10.818	39.845	11.665
14	91.321	3.571	29.847	720.277	7.704	54.855
15	126.669	12.213	103.732	111.549	17.956	104.924
16	13.046	40.223	38.267	19.614	30.191	12.835
17	156.362	11.135	47.746	30.927	19.873	22.534
18	36.423	2.383	36.831	32.305	4.024	34.951
19	29.599	74.556	41.624	24.343	14.505	17.974
20	100.592	28.286	32.057	82.921	22.182	30.395
21	54.916	2.544	34.071	57.404	6.303	44.927
22	14.712	36.305	77.680	11.368	39.749	63.817

Key: LFP = linearized Fisher-Pry model
LGZ = linearized Gompertz growth curve
LWB = linearized Weibull growth curve
PL = Pearl growth curve
GZ = Gompertz growth curve
WB = Weibull growth curve

TABLE 45. Mean forecast errors at 25% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	25.398	8.710	5.422	10.433	11.270	5.605
2	43.930	1.909	29.510	41.275	3.498	4.140
3	0.295	95.046	202.552	8.073	78.719	55.178
4	27.636	5.240	39.831	2.915	29.729	89.552
5	2.431	98.819	12.635	2.154	15.348	13.250
6	5.104	45.333	179.374	13.531	59.523	193.139
7	62.767	3.056	49.173	53.824	4.430	66.873
8	6.293	47.649	68.955	4.436	38.313	32.450
9	59.400	6.899	47.381	35.820	8.723	35.592
10	7.944	101.583	57.448	12.455	87.371	37.289
11	22.053	6.059	31.033	5.374	9.411	34.926
12	5.506	38.484	93.187	20.806	71.028	141.150
13	25.427	21.700	18.380	20.064	5.759	25.039
14	86.383	5.185	56.457	63.567	14.321	63.839
15	105.157	13.733	66.287	37.434	3.987	12.442
16	13.626	86.390	85.974	26.966	86.571	78.303
17	44.436	19.201	10.384	4.811	52.234	66.568
18	30.773	4.451	32.128	23.387	7.066	27.080
19	39.185	12.050	42.298	26.681	2.664	40.222
20	59.450	15.411	26.202	32.645	9.507	16.222
21	43.841	3.520	35.709	28.521	3.452	26.270
22	17.061	18.413	18.892	18.521	7.905	24.414

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

TABLE 46. Mean forecast errors at 50% penetration level

case	LFP	LGZ	models LWB	PL	GZ	WB
1	13.616	3.694	7.600	5.150	3.426	3.923
2	17.610	1.069	2.365	4.997	8.661	4.140
3	0.333	53.310	30.053	0.635	8.395	1.904
4	16.297	1.314	2.321	7.127	0.857	3.736
5	1.128	36.086	2.759	2.490	1.584	6.730
6	3.498	2.443	3.772	7.895	0.851	11.817
7	42.821	1.366	46.315	12.900	1.564	66.873
8	1.634	13.487	6.999	1.181	6.315	1.987
9	37.303	6.427	27.056	17.430	5.144	15.716
10	4.345	55.993	15.681	1.380	22.391	0.353
11	1.608	15.029	24.497	4.427	16.770	20.730
12	2.561	33.510	41.653	8.406	33.662	28.256
13	27.597	2.306	33.002	20.360	4.919	30.827
14	65.006	10.584	49.023	32.418	11.958	31.807
15	27.891	0.404	5.496	1.613	1.397	2.408
16	13.757	64.368	10.170	12.149	41.770	5.840
17	6.163	31.383	18.752	9.161	38.655	24.188
18	14.038	1.862	14.921	5.235	0.965	5.217
19	31.836	3.270	31.179	10.510	0.476	18.537
20	30.118	6.149	15.858	11.934	2.609	3.845
21	30.454	4.206	17.872	19.132	4.274	15.233
22	23.773	3.071	23.020	26.700	8.685	34.717

Key: LFP = linearized Fisher-Pry model
    LGZ = linearized Gompertz growth curve
    LWB = linearized Weibull growth curve
    PL = Pearl growth curve
    GZ = Gompertz growth curve
    WB = Weibull growth curve

TABLE 47. Mean forecast errors at 75% penetration level

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
1	1.138	4.020	0.807	1.138	3.688	1.645
2	4.691	0.946	1.387	1.613	1.278	1.298
3	0.124	18.541	0.431	0.128	2.155	1.439
4	7.262	0.834	1.421	3.360	0.823	3.055
5	1.274	19.568	2.618	1.341	1.617	3.917
6	1.112	0.146	1.634	1.090	0.204	1.451
7	10.288	1.239	8.414	1.163	1.254	1.268
8	1.061	0.879	1.666	1.841	0.205	3.487
9	15.894	3.617	12.248	7.694	2.729	5.821
10	1.693	21.151	1.865	0.427	5.570	0.244
11	3.171	16.053	14.586	6.108	16.565	10.219
12	4.703	26.414	20.942	7.895	24.124	10.157
13	20.545	1.568	23.373	12.581	4.536	16.388
14	26.666	4.273	20.157	10.006	3.563	5.564
15	12.653	0.694	0.779	2.589	0.318	0.379
16	12.002	45.046	29.407	7.814	24.356	4.633
17	2.597	24.481	16.350	6.147	21.566	8.538
18	3.956	0.816	5.161	1.941	0.588	2.202
19	17.968	0.390	20.238	5.108	0.490	9.730
20	9.867	2.696	8.642	5.707	2.296	3.845
21	5.165	0.545	2.642	3.038	0.717	0.926
22	18.843	4.499	20.487	15.571	7.603	17.173

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

APPENDIX H. RESULTS FOR NORMALITY OF ERROR TERMS

TABLE 48. Individual test results for normality of error terms

case	LFP	LGZ	models LWB	PL	GZ	WB
			a - levels			
1	0.48	0.39	0.67	0.60	0.09	0.35
2	0.12	0.29	0.01 *	0.38	0.01 *	0.55
3	0.02 *	0.30	0.22	0.09	0.04 *	0.31
4	0.01 *	0.01 *	0.01 *	0.03 *	0.15	0.01 *
5	0.01 *	0.23	0.01 *	0.28	0.08	0.02 *
6	0.44	0.01 *	0.01 *	0.01 *	0.36	0.03 *
7	0.01 *	0.18	0.01 *	0.01 *	0.08	0.01 *
8	0.06	0.01 *	0.01 *	0.01 *	0.01 *	0.01 *
9	0.01 *	0.07	0.04 *	0.02 *	0.04 *	0.07
10	0.01 *	0.01 *	0.44	0.10	0.35	0.09
11	0.96	0.06	0.01 *	0.16	0.26	0.08
12	0.01 *	0.01 *	0.01 *	0.01 *	0.01 *	0.01 *
13	0.03 *	0.58	0.02 *	0.02 *	0.44	0.23
14	0.35	0.32	0.39	0.06	0.35	0.59
15	0.49	0.99	0.01 *	0.45	0.74	0.83
16	0.08	0.01 *	0.03 *	0.03 *	0.02 *	0.06
17	0.23	0.06	0.02 *	0.02 *	0.13	0.52
18	0.01 *	0.01 *	0.06	0.71	0.04 *	0.38
19	0.13	0.42	0.01 *	0.11	0.23	0.54
20	0.01 *	0.61	0.06	0.02 *	0.01 *	0.01 *
21	0.15	0.48	0.02 *	0.46	0.13	0.42
22	0.20	0.06	0.50	0.01 *	0.06	0.09

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

* = Rejection occurs at the $a \leq 0.05$ level of significance

APPENDIX I.   RESULTS FOR HOMOSCEDASTICITY OF ERROR TERMS

TABLE 49.   Individual test results for homoscedasticity

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
			F - statistic ·			
1	1.14	8.43 *	1.96	1.63	1.03	1.28
2	3.13	1.70	45.79 *	3.77 *	14.86 *	9.26 *
3	4.11 *	2.17	4.22 *	3.11	1.94	1.40
4	6.34 *	2.48 *	11.64 *	1.43	1.04	1.10
5	2.92 *	5.86 *	5.57 *	1.44	1.19	1.01
6	2.69	3.56 *	30.43 *	13.39 *	2.44	9.37 *
7	1.39	6.67 *	17.90 *	59.91 *	30.97 *	41.69 *
8	1.69	2.12 *	43.60 *	29.55 *	38.69 *	38.30 *
9	79.40 *	1.28	42.37 *	16.40 *	1.95	4.42 *
10	24.15 *	50.60 *	30.57 *	1.58	1.20	9.27 *
11	1.62	4.99 *	2.99 *	6.91 *	9.97 *	5.49 *
12	2.21 *	5.52 *	13.38 *	1.16	2.91 *	1.40
13	1.66	1.07	32.80 *	35.01 *	3.62 *	8.24 *
14	199.15 *	9.35 *	209.02 *	17.73 *	10.14 *	16.59 *
15	12.54 *	1.49	17.44 *	1.42	1.01	1.24
16	1.97	24.31 *	4.44 *	6.82 *	13.06 *	3.44 *
17	11.52 *	29.45 *	3.19 *	15.81 *	7.94 *	3.64 *
18	1.85	18.36 *	5.78 *	106.99 *	16.25 *	31.95 *
19	32.40 *	5.84 *	235.50 *	3.10	4.73 *	5.14 *
20	21.50 *	1.12	16.08 *	20.23 *	6.91 *	7.12 *
21	1.40	34.00 *	3.25	8.49 *	18.14 *	15.94 *
22	2.06	1.88	59.51 *	1.04	1.02	1.18

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

     * = Rejection occurs when F(n1, n2, 0.05) < F-statistic

APPENDIX J.   RESULTS FOR NONAUTOCORRELATION

TABLE 50.   Individual test results for nonautocorrelation

			models			
case	LFP	LGZ	LWB	PL	GZ	WB
			Durbin-Watson statistic			
1	0.85 *	0.58 *	1.10 *	1.29	1.27	1.43
2	0.23 *	0.89 *	0.44 *	0.58 *	1.07 *	1.05 *
3	0.25 *	0.42 *	0.70 *	0.35 *	0.57 *	0.58 *
4	0.14 *	0.01 *	0.09 *	0.18 *	0.06 *	0.11 *
5	0.11 *	0.22 *	0.35 *	0.13 *	0.26. *	0.18 *
6	1.06 *	0.29 *	0.22 *	1.00 *	1.12 *	0.76 *
7	0.15 *	0.33 *	0.40 *	0.28 *	0.49 *	0.40 *
8	0.27 *	0.31 *	0.14 *	0.88 *	0.85 *	0.68 *
9	0.12 *	0.12 *	0.15 *	0.11 *	0.18 *	0.17 *
10	0.25 *	0.17 *	0.20 *	0.28 *	0.13 *	0.24 *
11	0.76 *	0.36 *	0.25 *	1.26 *	0.51 *	0.57 *
12	0.15 *	0.04 *	0.11 *	0.21 *	0.10 *	0.15 *
13	0.13 *	0.31 *	0.60 *	0.15 *	0.31 *	0.21 *
14	0.24 *	0.19 *	0.23 *	0.21 *	0.32 *	0.31 *
15	0.51 *	1.32	1.94	0.92 *	1.57	1.75
16	0.09 *	0.05 *	0.12 *	0.15 *	0.08 *	0.12 *
17	0.88 *	0.38 *	0.25 *	1.26 *	0.63 *	0.89 *
18	0.33 *	0.69 *	0.22 *	0.23 *	0.40 *	0.30 *
19	0.38 *	0.85 *	0.75 *	0.48 *	1.90	0.56 *
20	0.24 *	0.54 *	0.27 *	0.21 *	0.45 *	0.48 *
21	0.18 *	0.51 *	0.95 *	0.24 *	0.43 *	0.42 *
22	0.15 *	0.23 *	0.35 *	0.13 *	0.22 *	0.19 *

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     LWB = linearized Weibull growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve
     WB = Weibull growth curve

   * = Rejection occurs when D-W statistic < d_L or
                   when D-W statistic > $4-d_L$.

APPENDIX K. VARIANCE OF ERROR TERMS

TABLE 51. Individual variance of error terms

| case | models | | | |
	LFP	LGZ	PL	GZ
1	0.2292160	0.0525357	0.0029417	0.0024465
2	0.2975730	0.0130950	0.0010470	0.0005711
3	0.2554270	0.0293741	0.0017589	0.0010870
4	0.1008040	0.1355550	0.0002318	0.0005311
5	0.3204550	0.0367397	0.0012222	0.0006940
6	0.0877781	0.2976250	0.0006161	0.0005329
7	0.3864110	0.1371680	0.0013586	0.0007593
8	0.1274400	0.0747629	0.0014856	0.0014323
9	1.0046100	0.0357771	0.0024237	0.0006864
10	0.3841428	0.7633370	0.0001275	0.0010039
11	0.0529496	0.0349301	0.0011847	0.0021862
12	0.1442540	0.2119620	0.0015687	0.0035691
13	0.3668930	0.0224309	0.0026228	0.0010121
14	0.8804420	0.0320668	0.0034421	0.0011714
15	0.3385020	0.0166305	0.0016682	0.0009756
16	0.1112850	0.2066290	0.0009711	0.0026270
17	0.0941606	0.1649800	0.0015493	0.0028344
18	0.9171340	0.2163360	0.0010380	0.0002875
19	1.2202200	0.0240050	0.0011578	0.0003067
20	0.7623390	0.0550025	0.0027035	0.0010732
21	0.2310750	0.0338791	0.0035234	0.0018849
22	0.3100070	0.0362644	0.0040943	0.0019637

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

APPENDIX L. SCALE OR INTERCEPT PARAMETER

TABLE 52. Individual scale or intercept parameter

| | models | | | |
case	LFP	LGZ	PL	GZ
1	-4.484549	-2.468645	31.233786	7.213293
2	-5.533922	-2.157066	54.967555	8.886245
3	-3.086024	-1.443972	15.035815	4.219136
4	-5.649673	-2.499071	305.675021	35.444588
5	-3.897951	-1.805320	29.198135	6.613107
6	-6.545642	-3.064132	507.521821	49.148946
7	-2.482421	-1.329700	20.122877	5.484244
8	-2.915803	-1.615880	25.322259	6.221314
9	-4.161412	-1.753934	19.587539	5.255510
10	-5.222297	-2.886377	192.345570	26.500924
11	-3.216227	-1.446504	22.830711	4.605687
12	-3.909521	-1.962871	47.057550	8.228604
13	-5.109756	-2.046338	47.271740	9.094102
14	-5.115401	-1.940510	23.800895	5.694928
15	-3.901465	-1.604261	18.390183	4.339082
16	-4.451384	-2.095253	162.042527	20.037088
17	-3.611650	-1.938400	47.521007	9.327611
18	-2.829723	-1.410079	21.057380	5.342873
19	-8.152847	-2.735915	134.605244	17.638173
20	-2.988615	-1.344400	11.921693	3.867563
21	-3.279234	-1.465643	14.482676	4.192919
22	-3.722765	-1.616986	21.105145	5.526387

Key: LFP = linearized Fisher-Pry model
LGZ = linearized Gompertz growth curve
PL = Pearl growth curve
GZ = Gompertz growth curve

APPENDIX M.   SHAPE OR SLOPE PARAMETER

TABLE 53.   Individual shape or slope parameter

case	models			
	LFP	LGZ	PL	GZ
1	0.43870464	0.32435529	0.35119117	0.24990061
2	0.30972490	0.15167074	0.23189589	0.15219363
3	0.21321582	0.14209824	0.20522847	0.14345075
4	0.08287913	0.04822931	0.08460480	0.05951213
5	0.15703515	0.10083954	0.14976016	0.10385738
6	0.37132370	0.23263003	0.36374133	0.25422221
7	0.17360722	0.14034324	0.23119534	0.16590052
8	0.08564683	0.06685287	0.10161739	0.07145905
9	0.29242052	0.17844120	0.25784213	0.18295178
10	0.42054041	0.30112853	0.39113670	0.27792421
11	0.09445175	0.05585878	0.09072801	0.05625309
12	0.05832573	0.03886742	0.05423985	0.03613516
13	0.20193767	0.15153638	0.24053554	0.16546877
14	0.45916711	0.22655788	0.31506374	0.21729011
15	0.27655397	0.14776453	0.21243174	0.13835071
16	0.18485122	0.11238403	0.20131800	0.13632655
17	0.19448020	0.13963656	0.19991890	0.13914457
18	0.33333540	0.25851340	0.45430408	0.31692628
19	0.39927461	0.16936198	0.26180950	0.17760933
20	0.22075314	0.15226814	0.22717380	0.16471168
21	0.20621697	0.12744141	0.18226587	0.12795654
22	0.23831202	0.14188159	0.21948786	0.15494606

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

APPENDIX N.   MEAN ESTIMATE ERRORS FOR TELEPHONE DATA

TABLE 54.   Mean estimate errors for telephone data

Company	model			
	LFP	LGZ	PL	GZ
1	1.2837	9.5399	0.8790	2.2203
2	6.1337	0.9857	2.9605	0.9036
3	2.6101	2.9804	1.2705	2.4409
4	8.1380	2.2709	4.1044	1.8304
5	3.8114	6.5298	3.0456	6.2336
6	10.4270	2.8495	1.1966	0.4199
7	0.9313	21.7421	0.7362	0.9867
8	10.6265	10.1376	2.0702	5.5130
10	21.4708	1.2645	2.9117	1.2100

Key: LFP = linearized Fisher-Pry model
     LGZ = linearized Gompertz growth curve
     PL = Pearl growth curve
     GZ = Gompertz growth curve

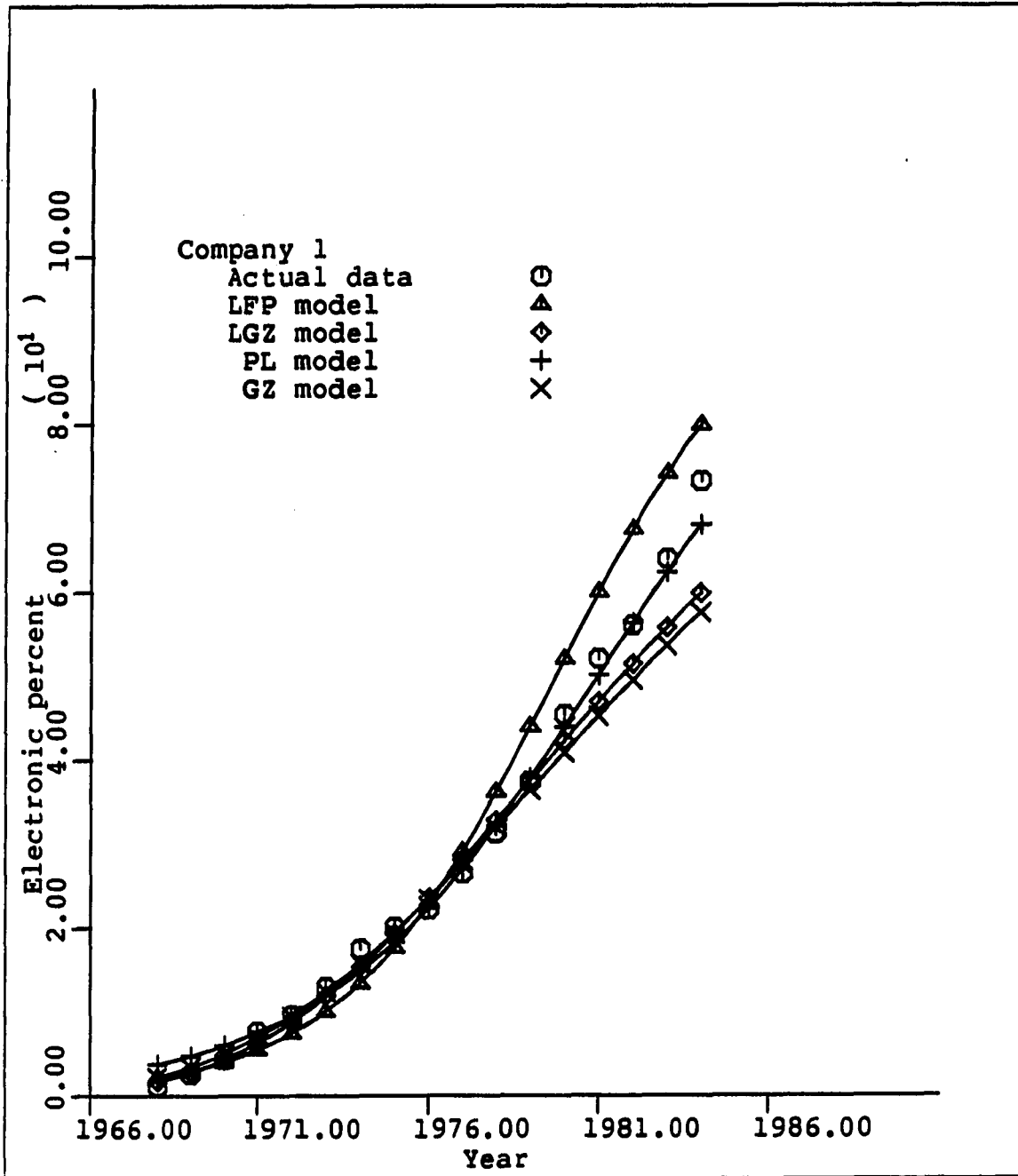**APPENDIX O.   ACTUAL VS FORECASTED WITH 4 MODELS**

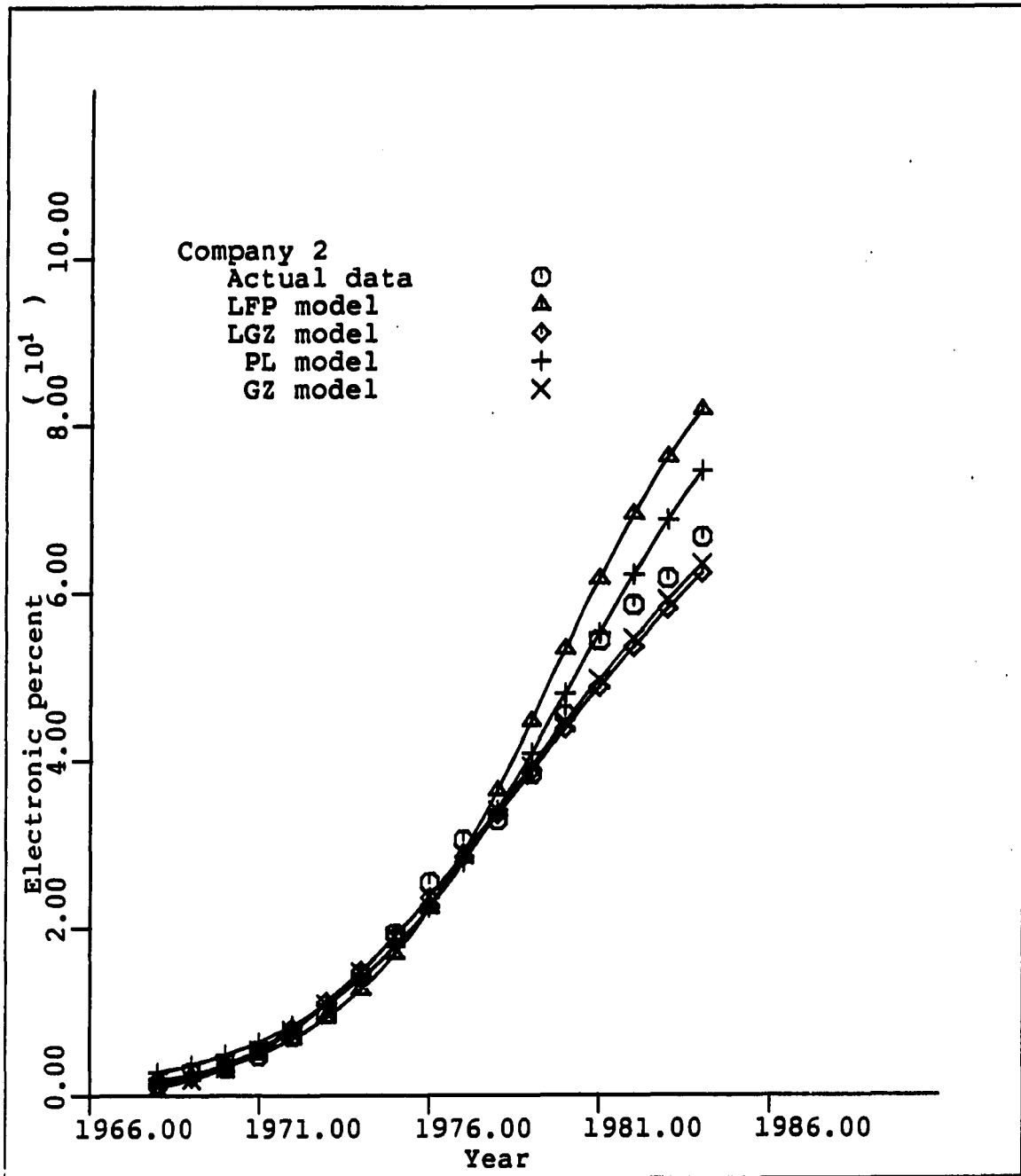FIGURE 18.   Actual vs forecasted for Company 1
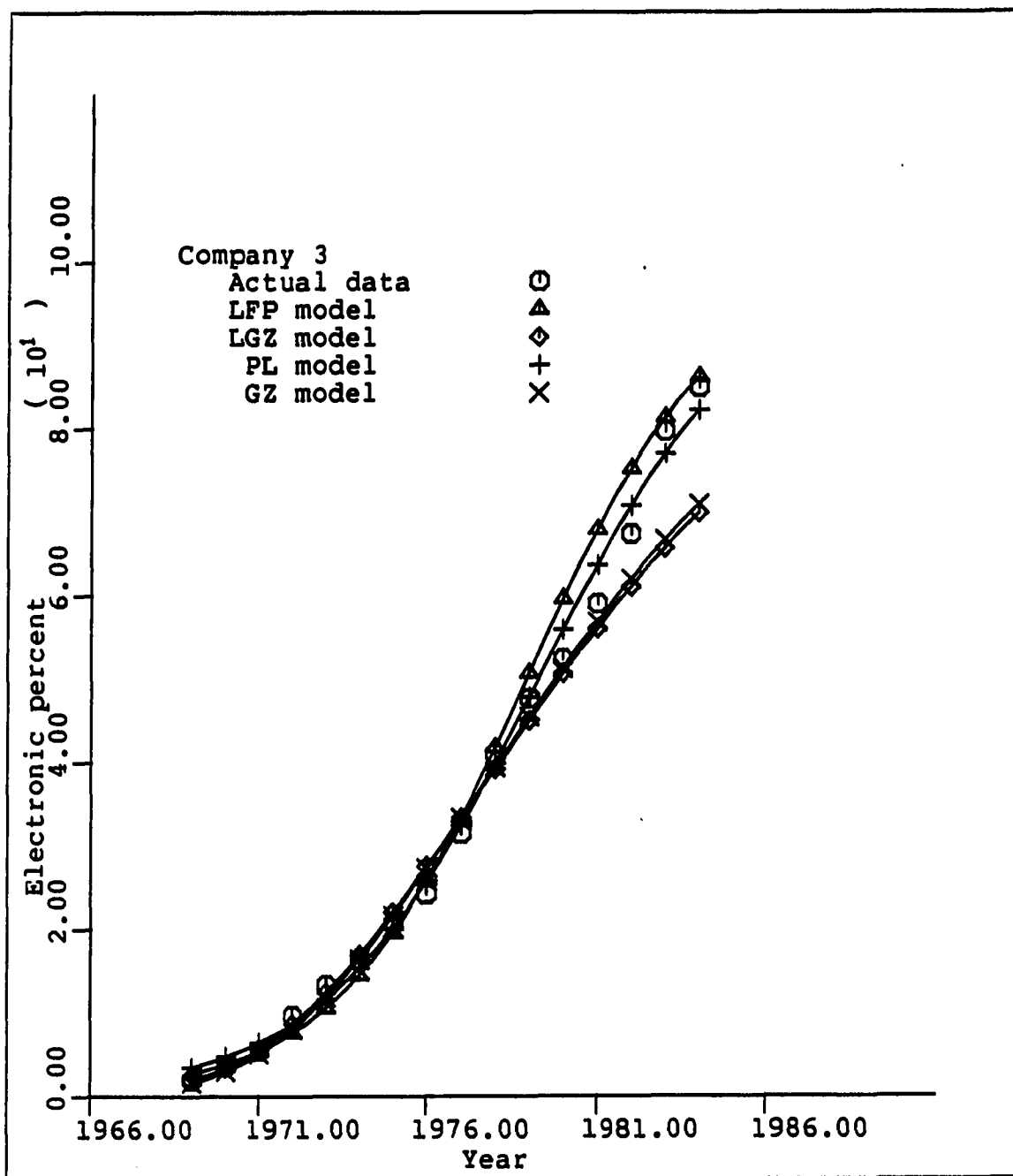
FIGURE 19. Actual vs forecasted for Company 2

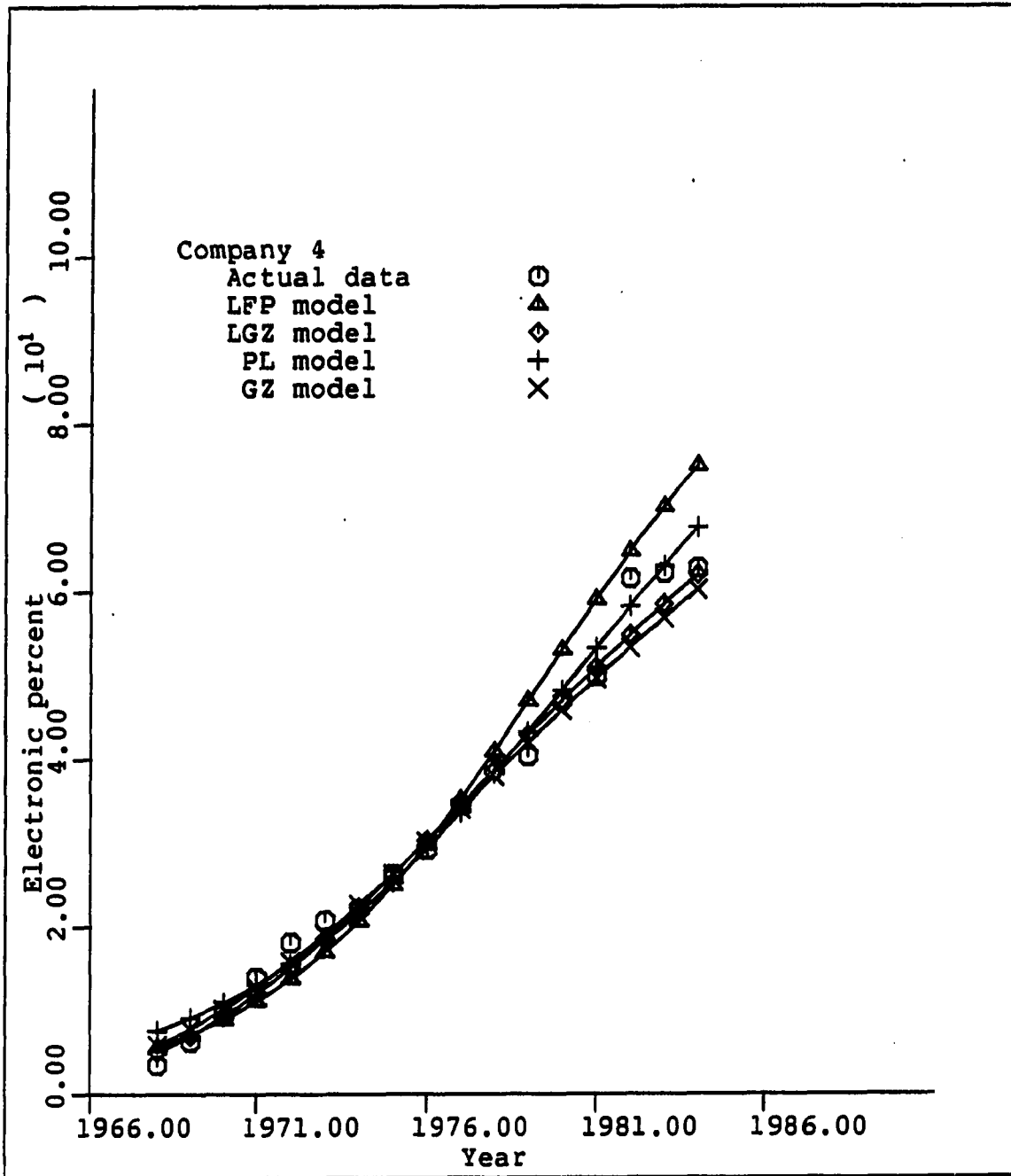FIGURE 20. Actual vs forecasted for Company 3

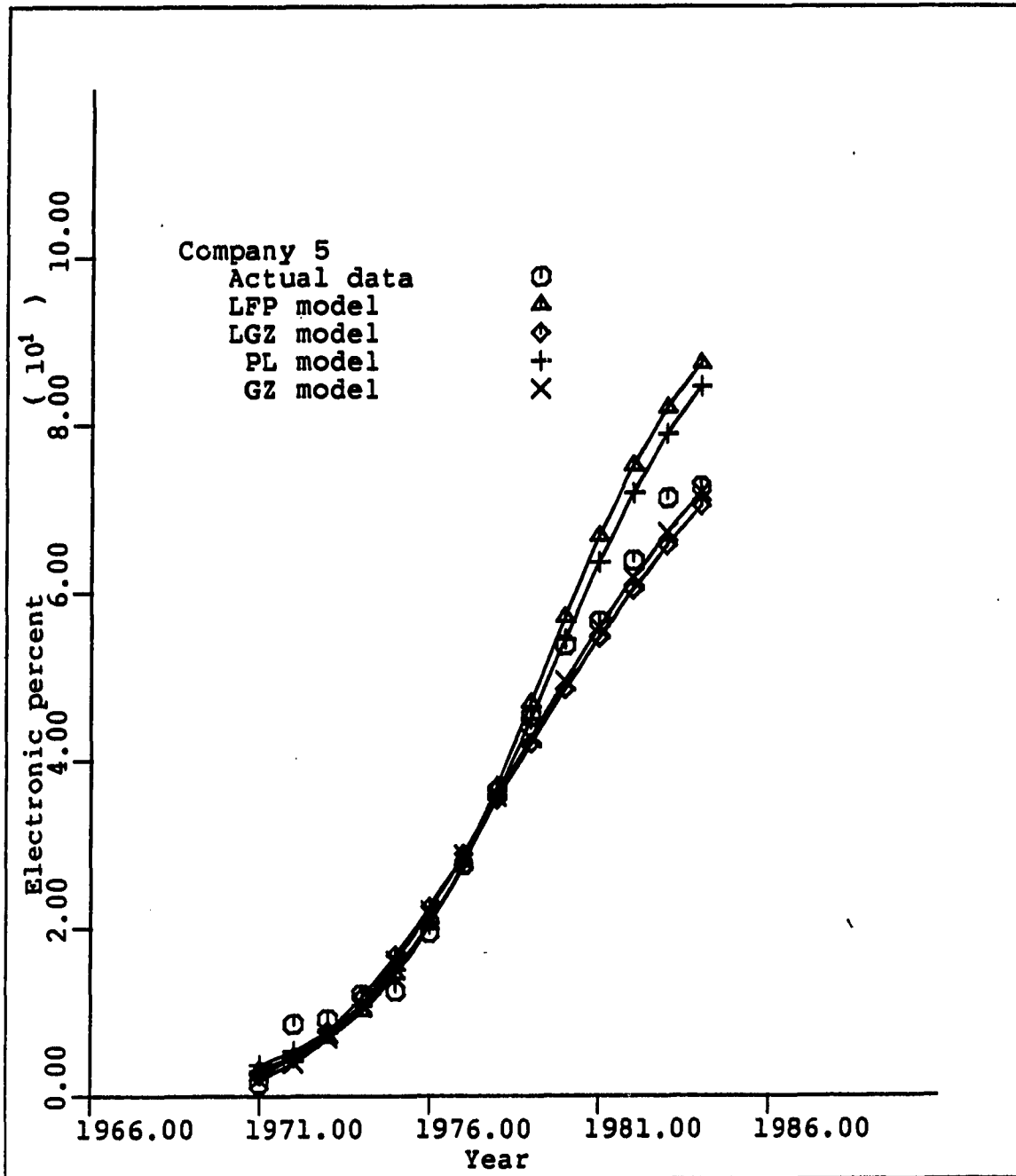FIGURE 21. Actual vs forecasted for Company 4

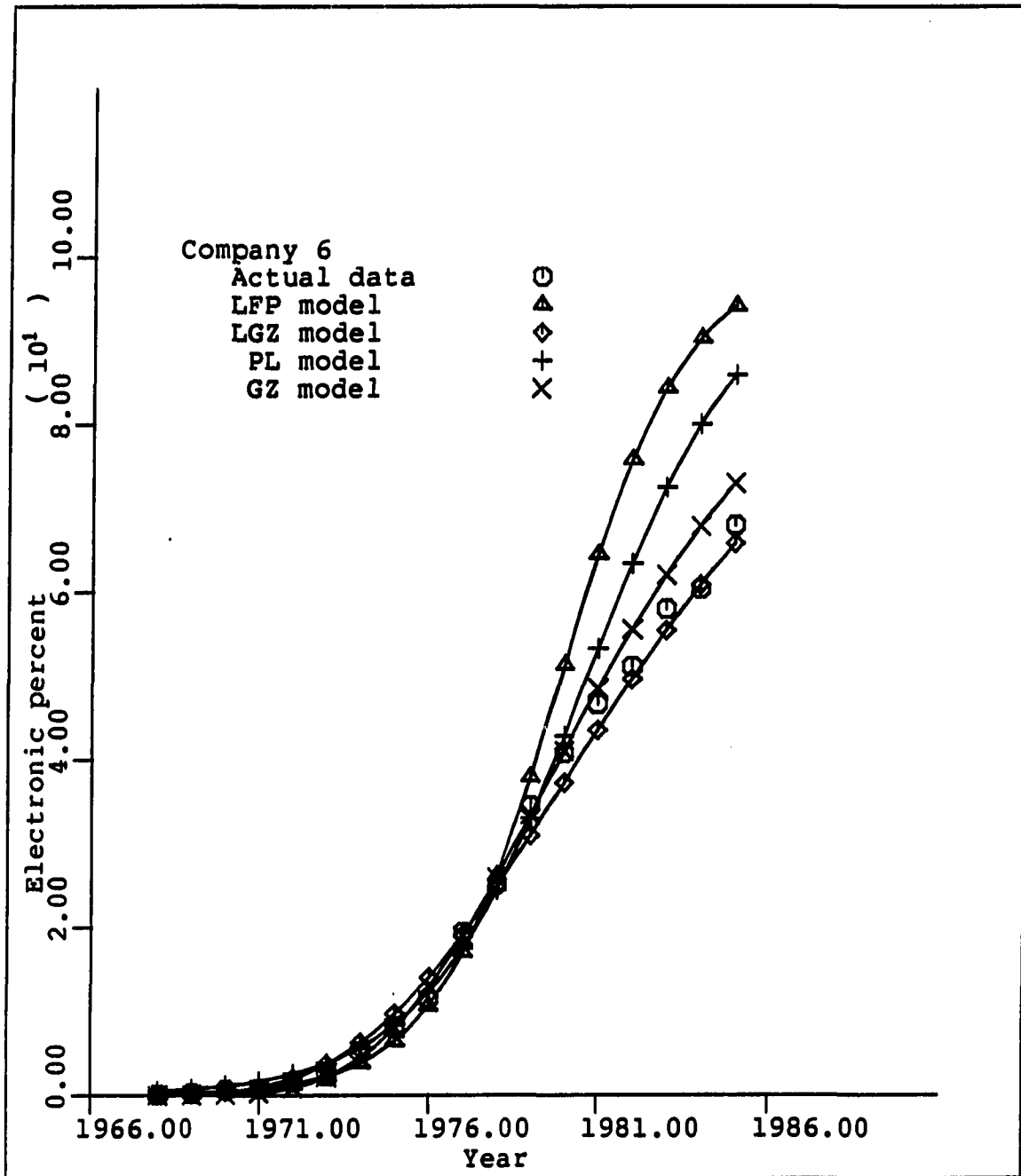FIGURE 22. Actual vs forecasted for Company 5

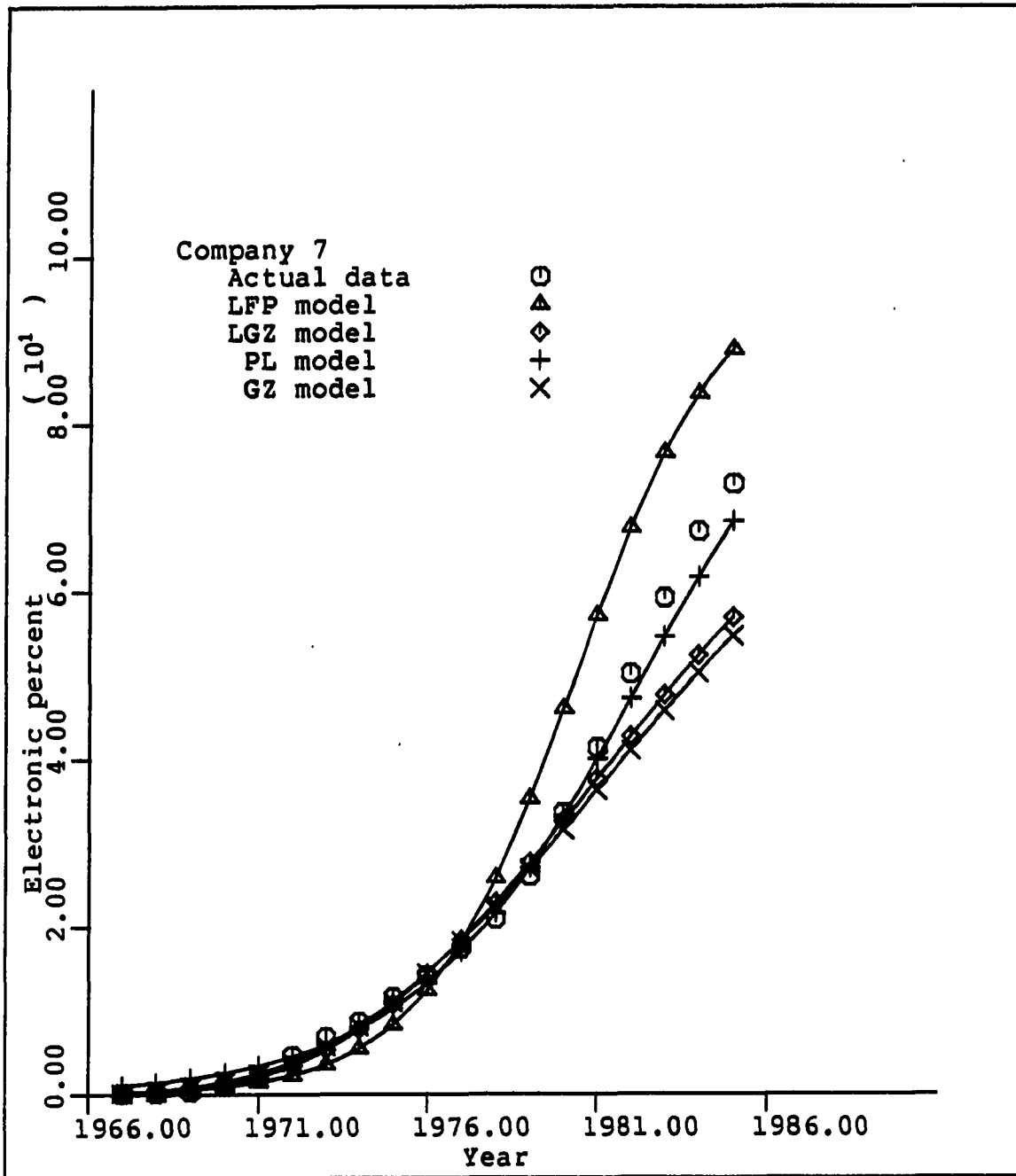FIGURE 23. Actual vs forecasted for Company 6

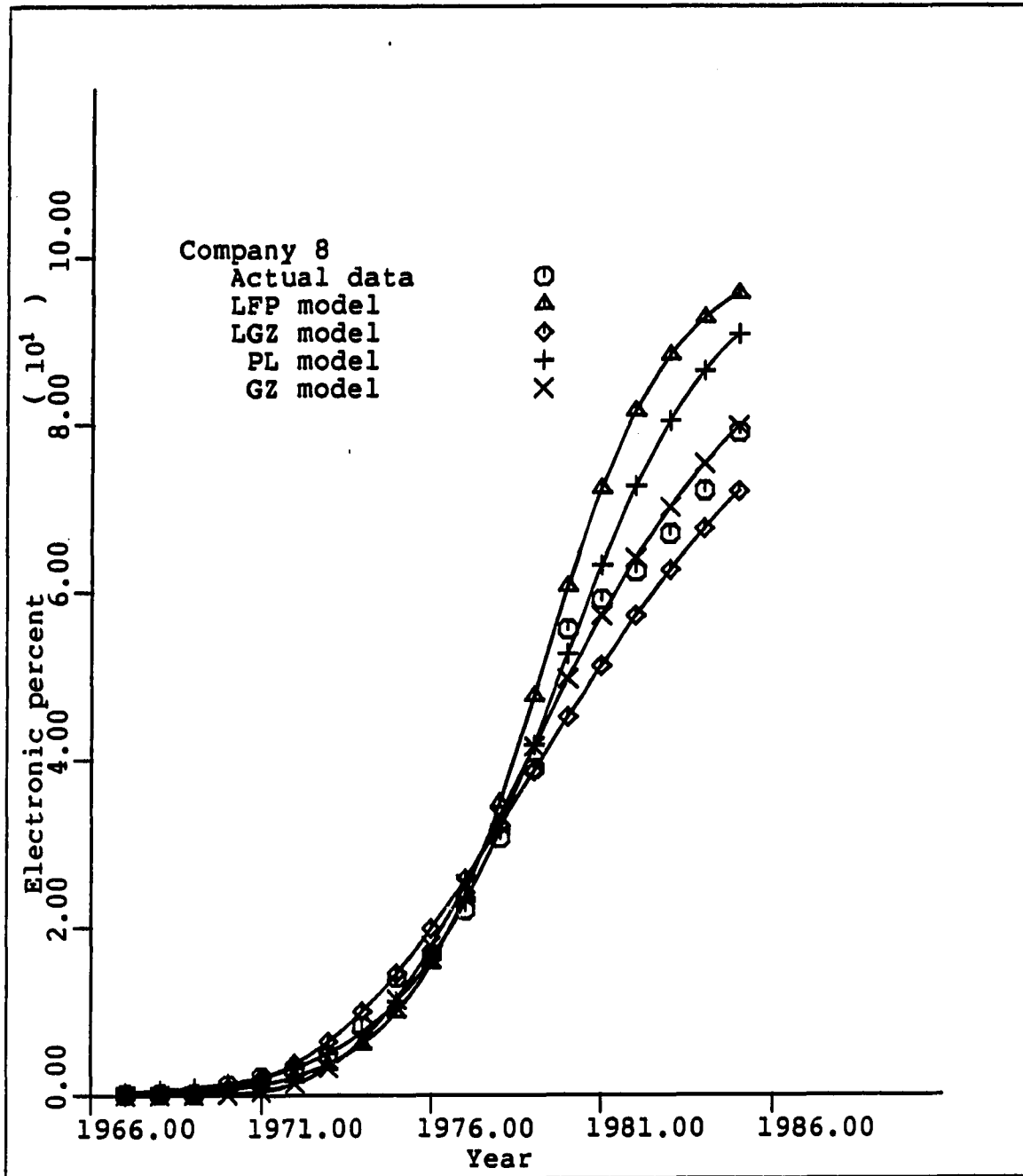FIGURE 24. Actual vs forecasted for Company 7
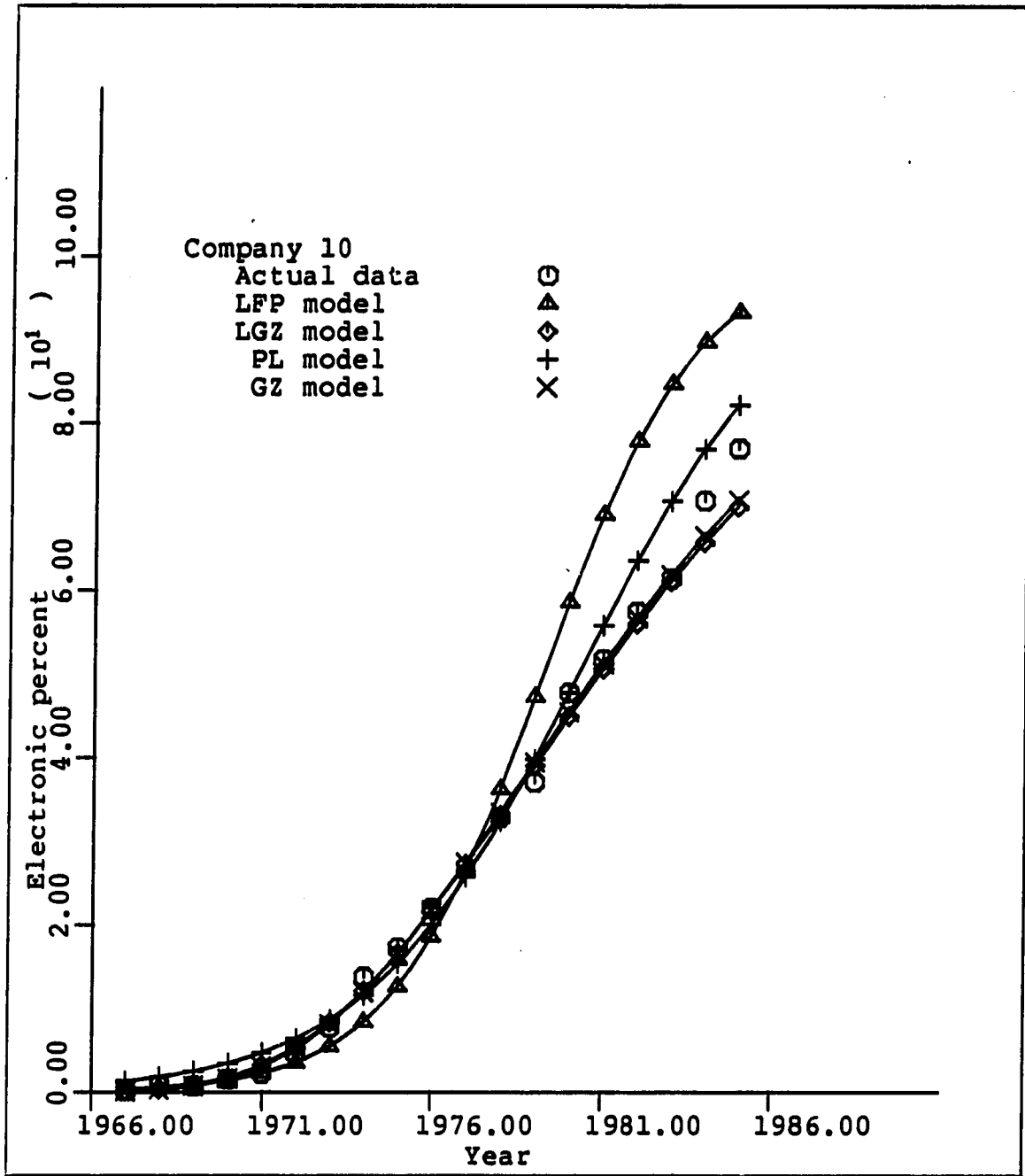
FIGURE 25. Actual vs forecasted for Company 8

FIGURE 26. Actual vs forecasted for Company 10

APPENDIX P.   BEST MODEL WITH DISCOUNT FACTORS

TABLE 55.   Best forecasting model for Company 1

		discount factor				
year	actual	1.00	0.95	0.90	0.80	0.70
1980	45.38	43.91	44.04	44.32	45.33	47.01
1981	52.08	50.13	50.85	51.77	54.30	57.81
1982	56.08	56.34	57.63	59.15	63.00	67.92
1983	63.97	62.36	64.13	66.14	70.93	76.59
1984	73.25	68.02	70.15	72.49	77.76	83.48
mean forecast error		7.20	3.07*	3.19	24.29	87.90
	parameter a	32.9024	44.4308	61.5051	129.5834	322.1856
	parameter b	0.2499	0.2734	0.2993	0.3598	0.4350

* indicates the smallest mean forecast error

TABLE 56.   Best forecasting model for Company 2

		discount factor				
year	actual	1.00	0.95	0.90	0.80	0.70
1980	45.39	44.50	44.25	44.09	44.08	44.52
1981	54.28	49.60	49.65	49.83	50.60	51.98
1982	58.52	54.48	54.81	55.30	56.75	58.91
1983	61.66	59.09	59.67	60.42	62.43	65.18
1984	66.60	63.40	64.19	65.14	67.58	70.75
mean forecast error		11.20	9.27	7.12	3.99*	7.16
	parameter G	5.2417	5.9076	6.7183	8.9925	12.7936
	parameter k	0.1436	0.1523	0.1619	0.1843	0.2124

* indicates the smallest mean forecast error

TABLE 57. Best forecasting model for Company 3

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1980	52.39	55.89	56.29	56.81	58.25	60.19
1981	59.00	63.63	64.65	65.82	68.66	72.07
1982	67.35	70.75	72.21	73.82	77.47	81.49
1983	79.67	76.97	78.68	80.51	84.37	88.25
1984	84.90	82.20	83.98	85.81	89.44	92.76
mean forecast error		11.99*	14.51	21.92	54.56	113.41
parameter a		38.2041	52.4425	73.8497	160.4236	402.8773
parameter b		0.3232	0.3510	0.3813	0.4509	0.5343

* indicates the smallest mean forecast error

TABLE 58. Best forecasting model for Company 4

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1980	46.99	45.86	45.28	44.82	44.55	45.24
1981	49.91	49.65	49.38	49.29	49.84	51.63
1982	61.55	53.33	53.36	53.59	54.91	57.64
1983	62.23	56.86	57.17	57.70	59.68	63.18
1984	62.80	60.23	60.79	61.58	64.11	68.20
mean forecast error		20.91	20.03	18.10	11.68	10.29*
parameter G		3.1570	3.5987	4.1288	5.6521	8.4772
parameter k		0.1078	0.1163	0.1260	0.1496	0.1822

* indicates the smallest mean forecast error

TABLE 59. Best forecasting model for Company 5

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1980	53.71	54.41	55.12	55.90	57.66	59.68
1981	56.53	63.59	64.97	66.43	69.59	72.99
1982	63.80	71.88	73.69	75.55	79.36	83.16
1983	71.24	78.91	80.88	82.83	86.59	90.02
1984	72.56	84.56	86.47	88.28	91.56	94.27
mean forecast error		63.73	91.46	124.49	204.94	301.06
	parameter a	37.8484	50.2438	67.8865	131.4717	278.9968
	parameter b	0.3810	0.4122	0.4455	0.5188	0.6023

* indicates the smallest mean forecast error

TABLE 60. Best forecasting model for Company 6

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1981	46.66	48.51	48.58	48.73	49.27	50.13
1982	51.13	55.57	55.98	56.49	57.83	59.63
1983	57.99	62.05	62.73	63.53	65.47	67.90
1984	60.39	67.87	68.75	69.74	72.06	74.84
1985	67.96	72.99	73.99	75.10	77.61	80.50
mean forecast error		24.14*	31.17	40.38	67.38	109.67
	parameter G	13.3063	15.3830	18.0060	25.7862	39.7327
	parameter k	0.2080	0.2185	0.2300	0.2568	0.2895

* indicates the smallest mean forecast error

TABLE 61. Best forecasting model for Company 7

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1981	41.37	40.03	40.34	40.80	42.15	44.10
1982	50.19	47.25	48.18	49.33	52.32	56.25
1983	59.22	54.59	56.11	57.89	62.29	67.69
1984	67.20	61.73	63.73	66.01	71.32	77.34
1985	72.87	68.40	70.73	73.29	78.93	84.76
mean forecast error		16.35	6.28	0.88*	13.64	71.99
parameter a		123.3889	175.2086	257.9886	636.1495	922.7729
parameter b		0.2941	0.3183	0.3454	0.4093	0.4883

* indicates the smallest mean forecast error

TABLE 62. Best forecasting model for Company 8

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1981	59.05	57.18	57.62	58.21	59.87	62.12
1982	62.43	64.03	64.88	65.92	68.56	71.87
1983	66.90	70.07	71.21	72.54	75.75	79.52
1984	72.05	75.30	76.61	78.09	81.52	85.30
1985	79.02	79.74	81.13	82.66	86.04	89.56
mean forecast error		5.42*	10.37	18.88	51.11	108.86
parameter G		16.5745	20.9021	27.1574	51.1629	114.6886
parameter k		0.2260	0.2423	0.2610	0.3068	0.3656

* indicates the smallest mean forecast error

TABLE 63. Best forecasting model for Company 10

year	actual	discount factor				
		1.00	0.95	0.90	0.80	0.70
1981	51.74	51.26	50.96	50.84	51.17	52.33
1982	57.44	56.75	56.77	57.00	58.20	60.46
1983	61.41	61.86	62.15	62.69	64.59	67.64
1984	70.74	66.55	67.07	67.84	70.25	73.81
1985	76.95	70.80	71.49	72.45	75.18	78.98
mean forecast error		11.26	8.98	6.26	2.87*	12.38
parameter G		7.9445	9.2200	10.9088	16.4715	28.5369
parameter k		0.1650	0.1744	0.1854	0.2135	0.2524

* indicates the smallest mean forecast error

APPENDIX Q.   FORECASTED WITH A DISCOUNT FACTOR
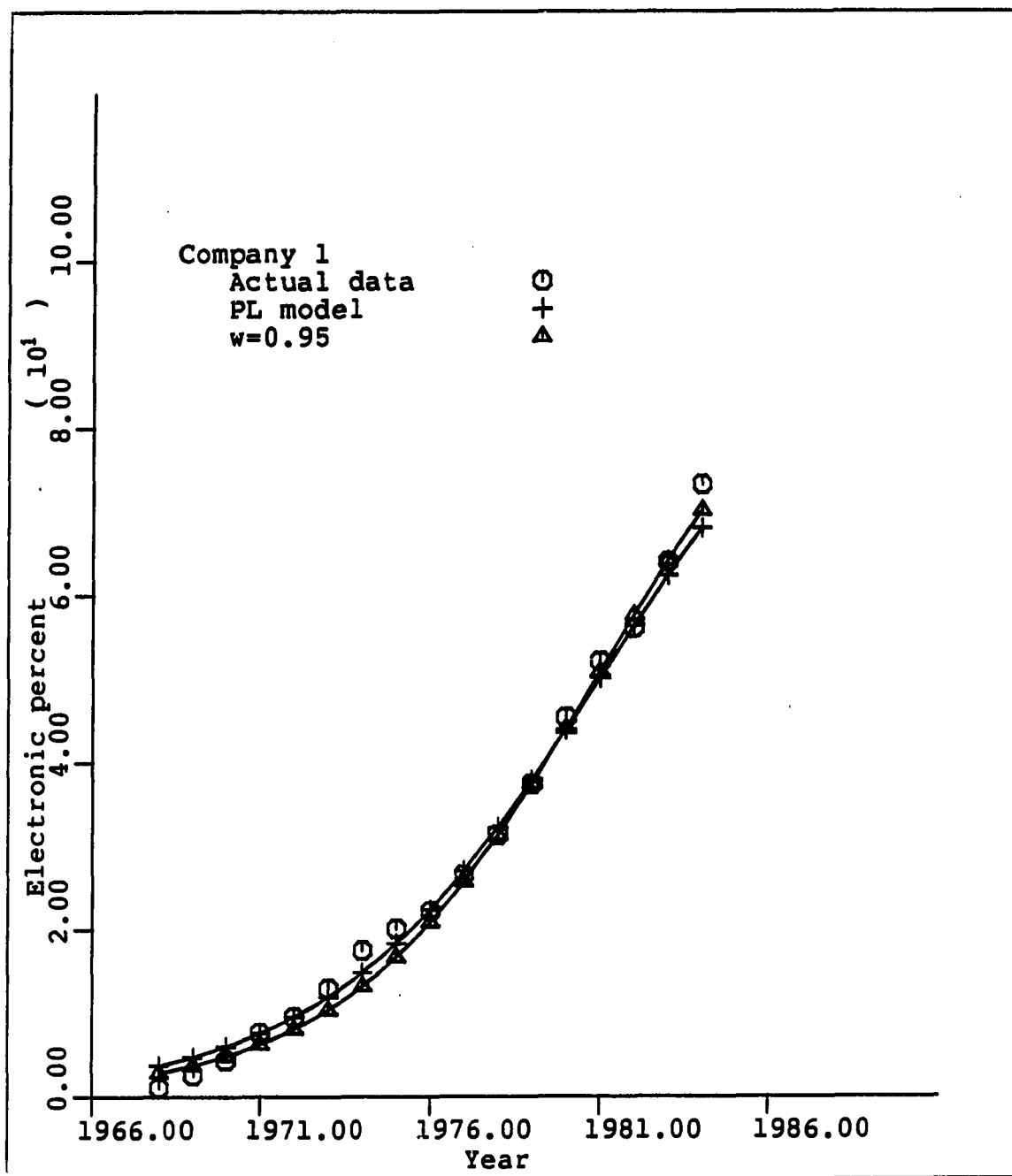
FIGURE 27.   Forecasted with w=0.95 for Company 1
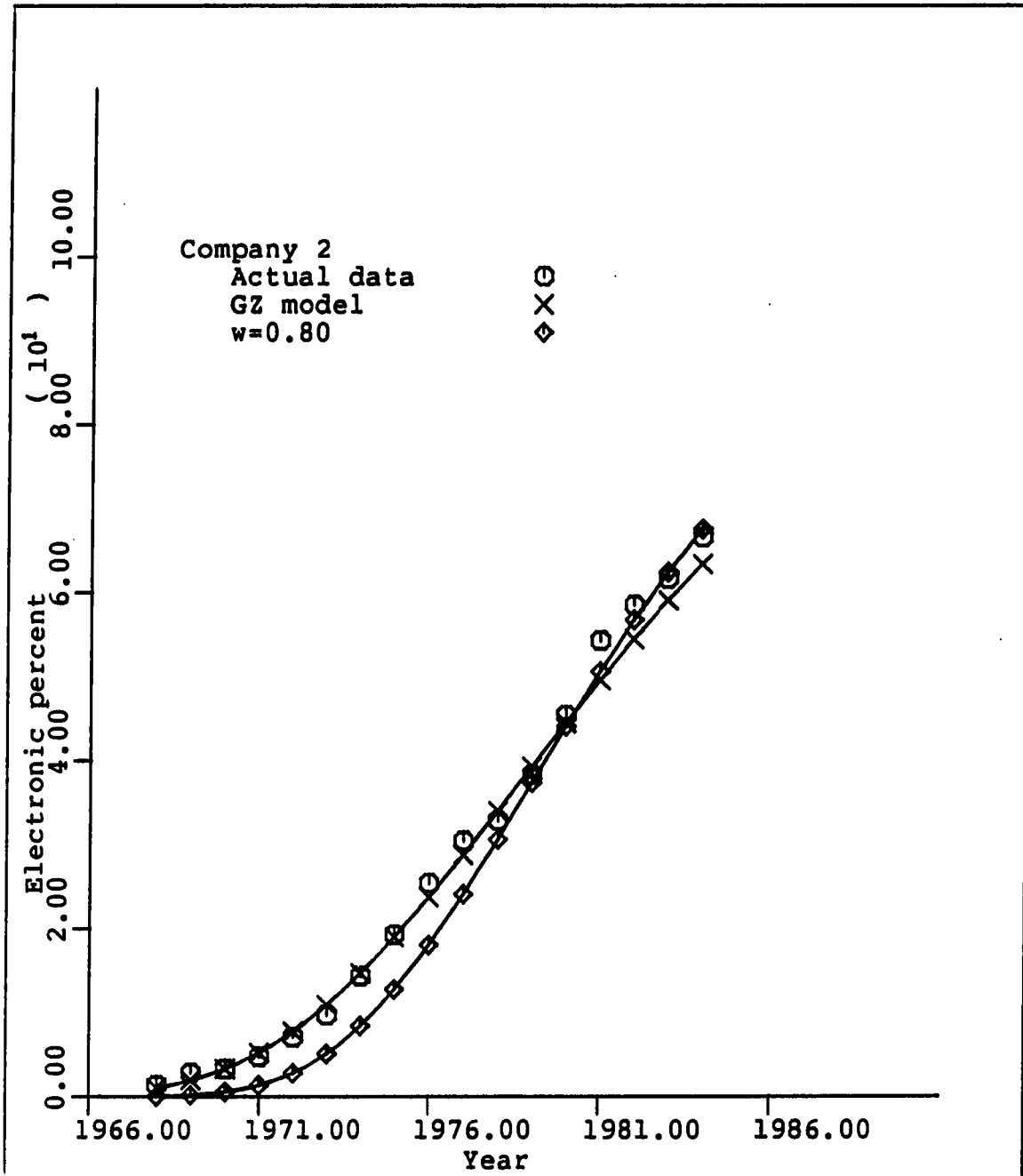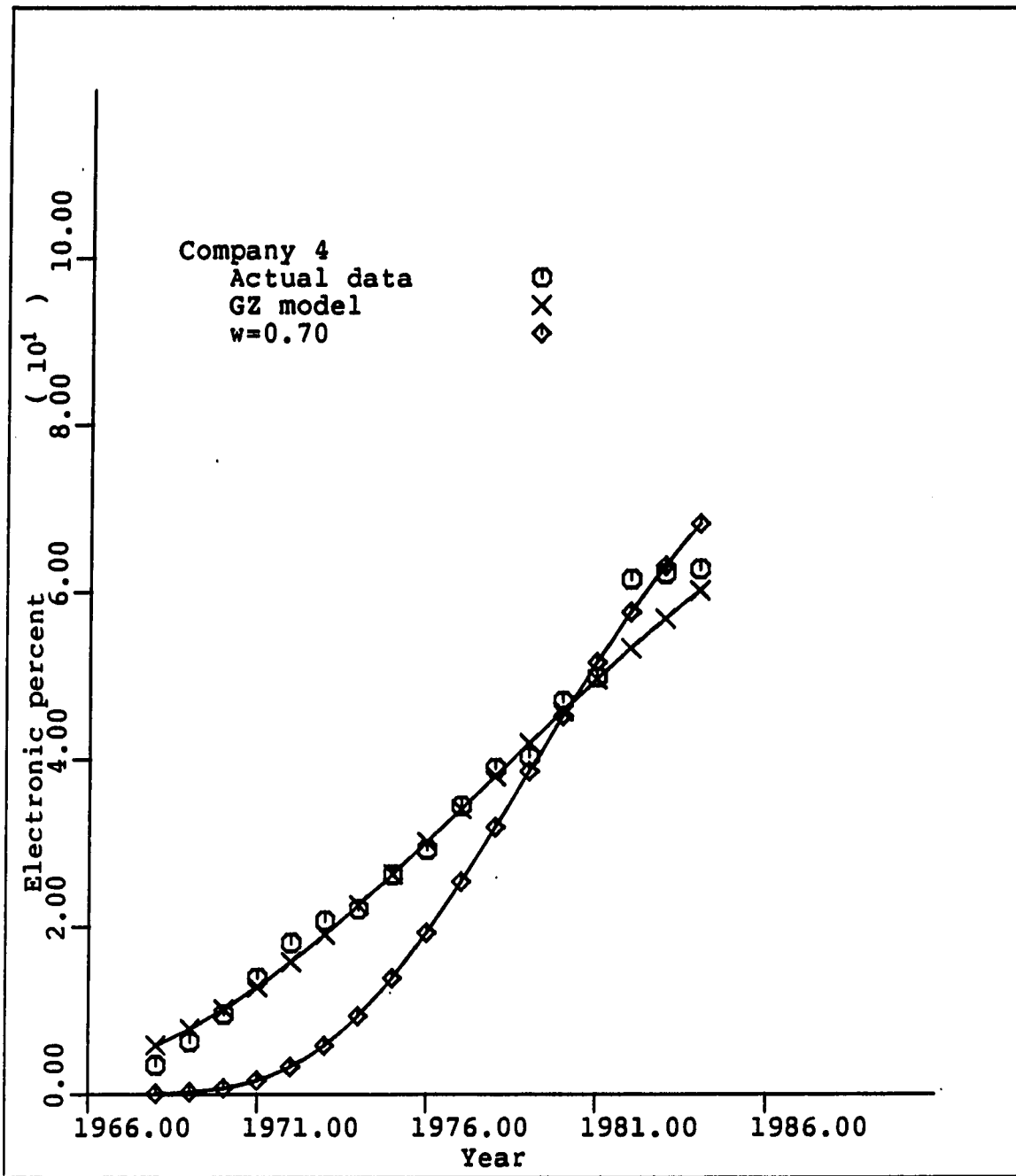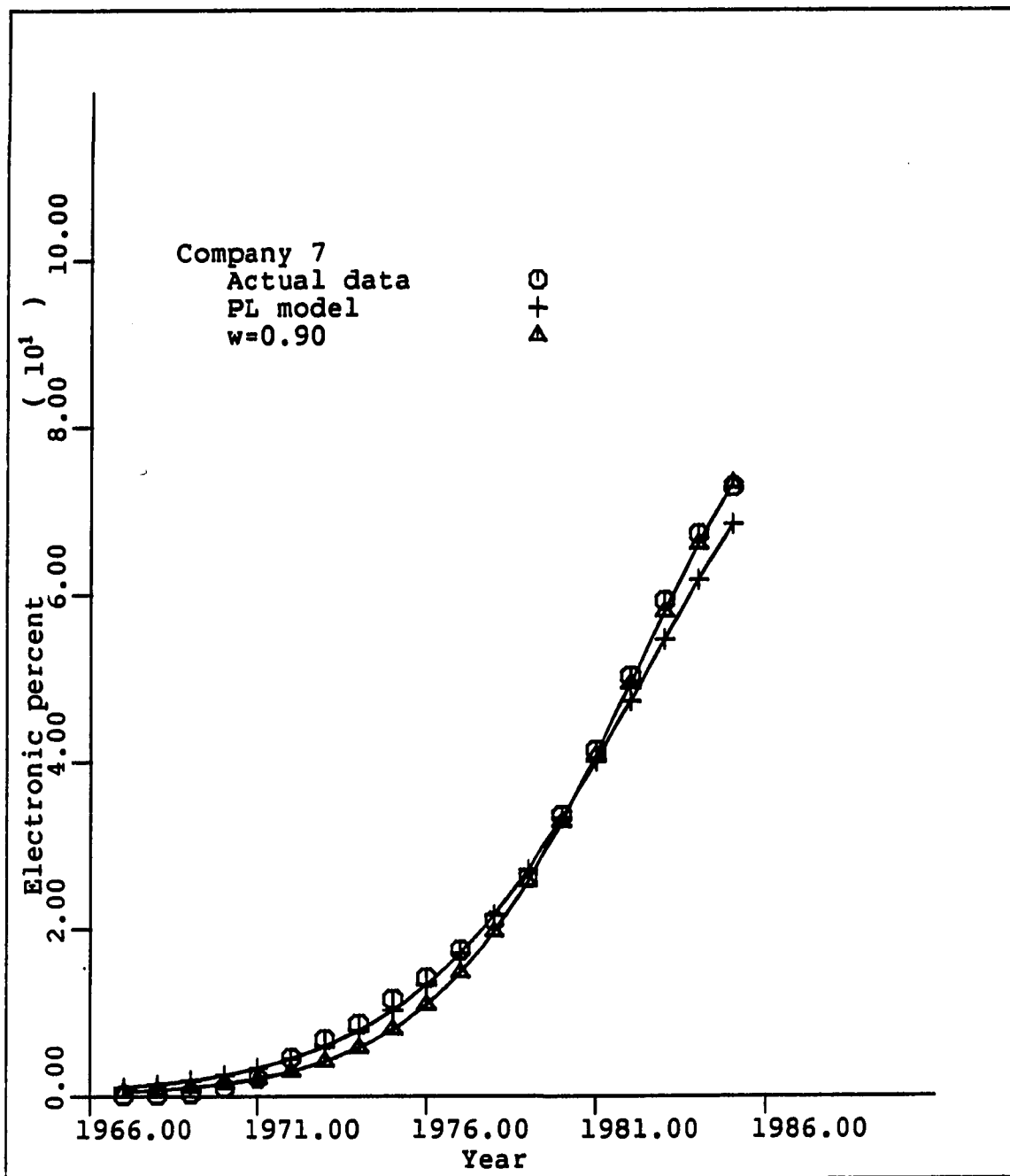
FIGURE 28. Forecasted with w=0.80 for Company 2

FIGURE 29. Forecasted with w=0.70 for Company 4
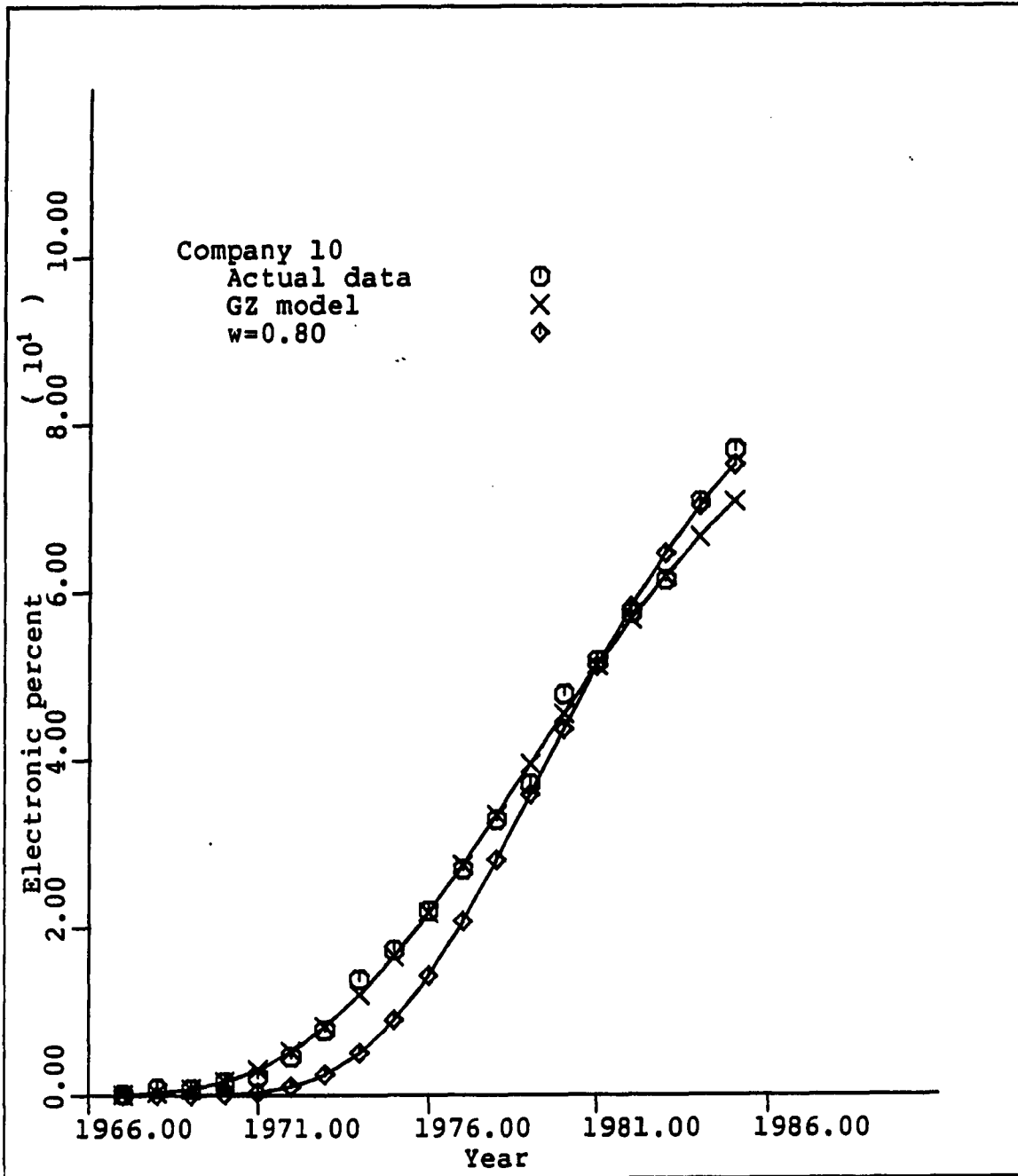
FIGURE 30. Forecasted with w=0.90 for Company 7

FIGURE 31.   Forecasted with w=0.80 for Company 10